Introduction to Cryptography

Lecture 6

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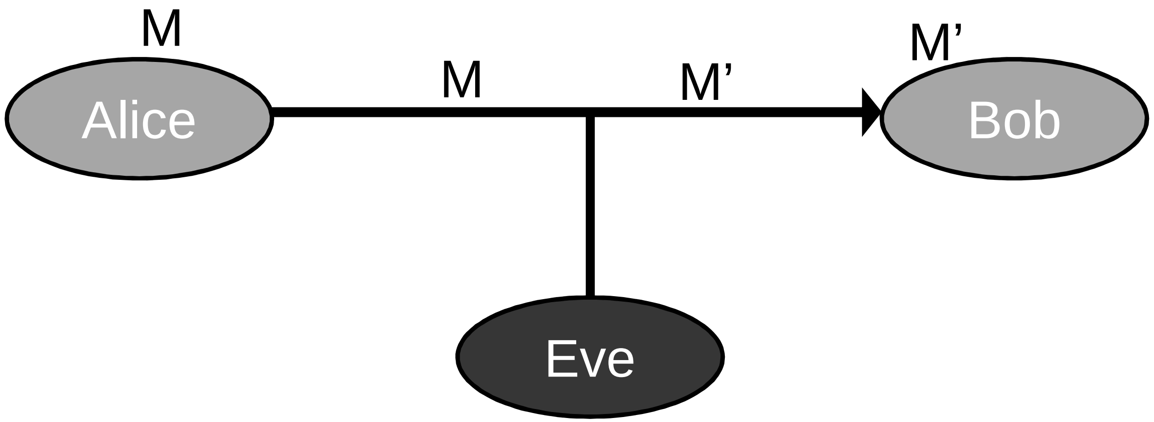
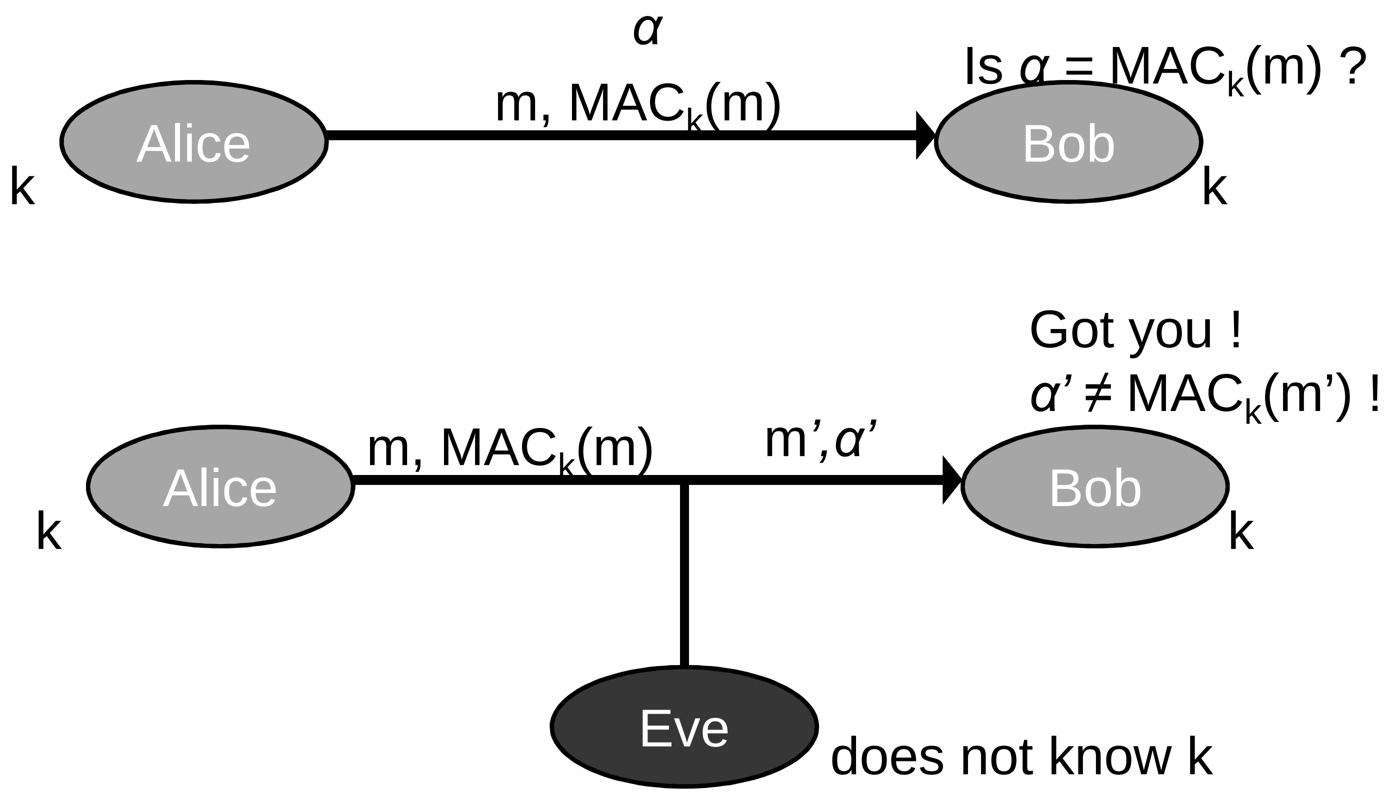
Data Integrity, Message Authentication

- Risk: an *active* adversary might change messages exchanged between Alice and Bob



• Authentication is orthogonal to secrecy. It is a relevant challenge regardless of whether encryption is applied.

Common Usage of MACs for message authentication



Requirements

- Security: The adversary,
  - Knows the MAC algorithm (but not K).
  - Is given many pairs $(m_i, MAC_K(m_i))$, where the m_i values might also be chosen by the adversary (chosen plaintext).
  - Cannot compute $(m, MAC_K(m))$ for any new m $(\forall i\ m \neq m_i)$.
  - The adversary must not be able to compute $MAC_K(m)$ *even* for a message m which is “meaningless” (since we don’t know the context of the attack).

- Efficiency: MAC output must be of fixed length, and as short as possible.
  - $\Rightarrow$ The MAC function is not 1-to-1.
  - $\Rightarrow$ An n bit MAC can be broken with prob. of at least 2^{-n}.

Constructing MACs

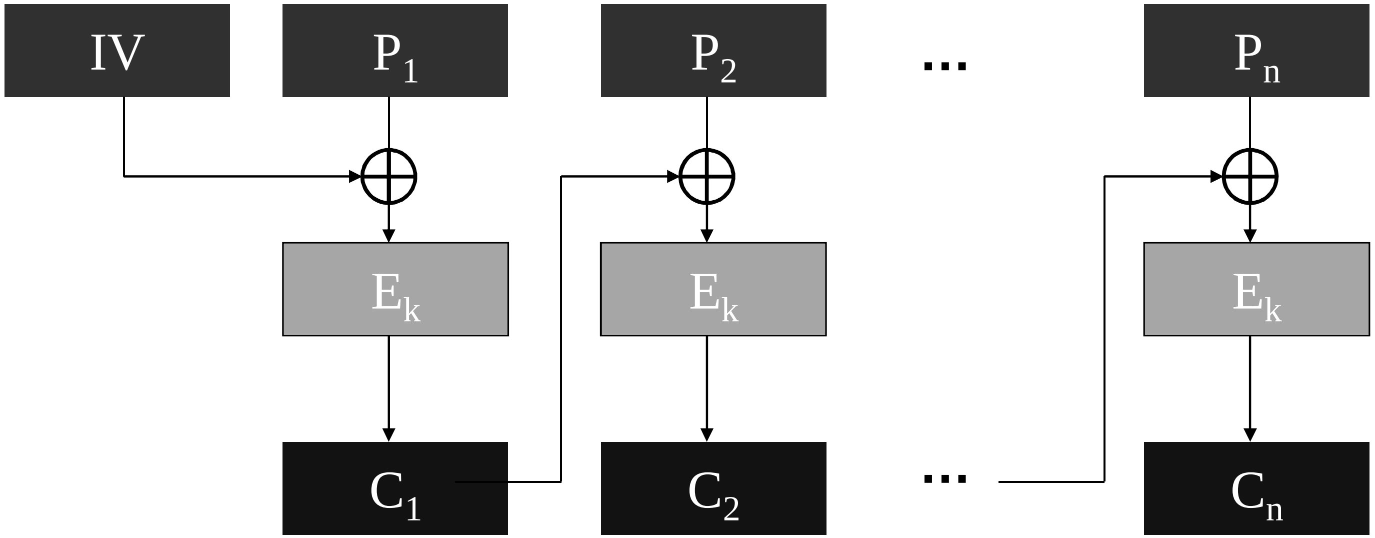
- Length of MAC output must be at least n bits, if we do not want the cheating probability to be greater than 2^{-n}

- Constructions of MACs
  - Based on block ciphers (CBC-MAC)

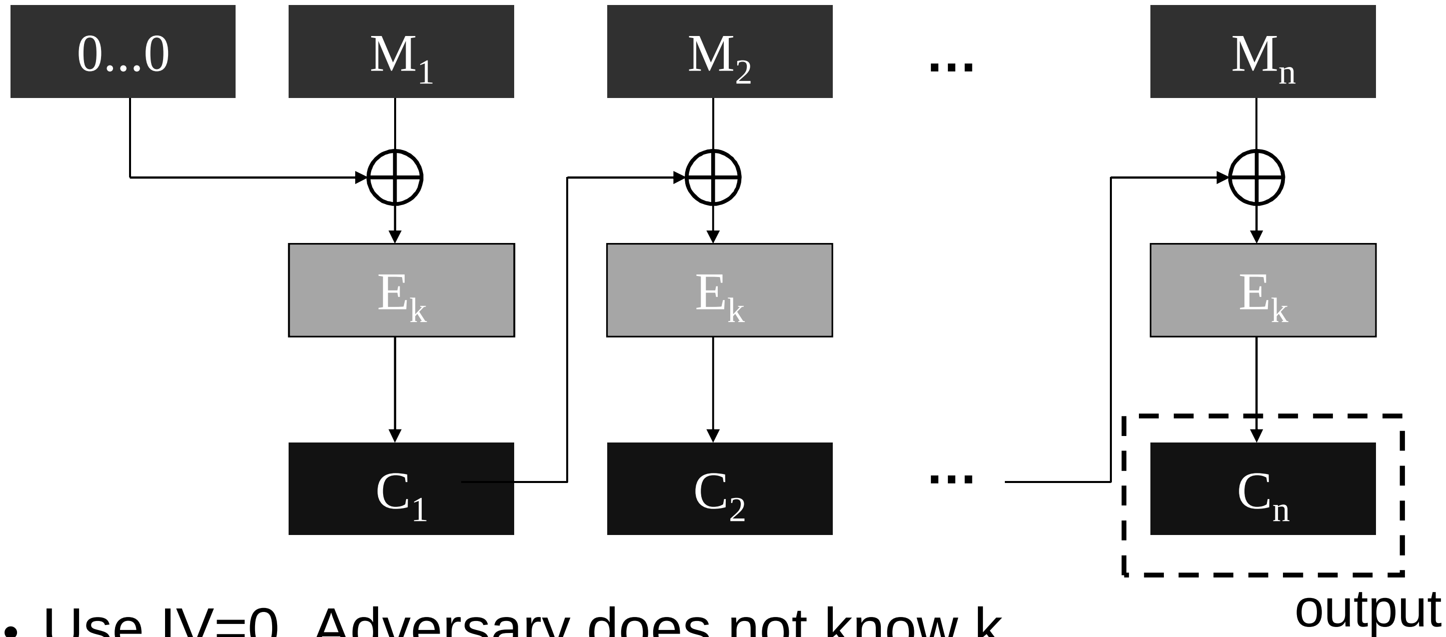
  or,
  - Based on hash functions
    - More efficient
    - At the time, encryption technology was controlled (export restricted) and it was preferable to use other means when possible.

CBC

- Reminder: CBC encryption
- Plaintext block is xored with previous ciphertext block



CBC MAC



- Use IV=0. Adversary does not know k.
- Encrypt M in CBC mode, using the MAC key. Discard $C_1,\ldots,C_{n-1}$ and define $MAC_K(M_1,\ldots,M_n)=C_n$.

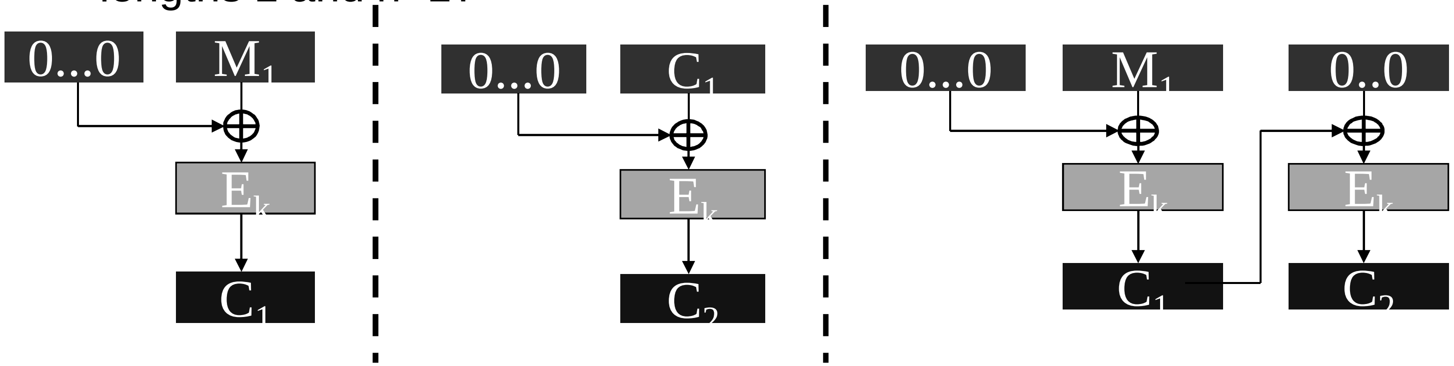
Security of CBC-MAC

- Claim: if E_K is pseudo-random then
  - CBC-MAC, applied to *fixed length messages*, is a pseudo-random function,
  - and is therefore a secure MAC (i.e., resilient to forgery).
- We will not prove this claim.

- But, CBC-MAC is insecure if variable lengths messages are allowed

Security of CBC-MAC

- Insecurity of CBC-MAC when applied to messages of variable length:
  - Get C_1 = CBC-MAC$_K(M_1) = E_K(0 \oplus M_1)$
  - Ask for MAC of C_1, i.e., C_2 = CBC-MAC$_K(C_1) = E_K(0 \oplus C_1)$
  - But, $E_K(C_1 \oplus 0) = E_K(E_K(0 \oplus M_1) \oplus 0)$ = CBC-MAC$_K(M_1 | 0)$
    - It's known that CBC-MAC is secure if message space is prefix-free.
    - Can you show, for every n, a collision between two messages of lengths 1 and n+1?



CBC-MAC for variable length messages

- Solution 1: The first block of the message is set to be its length. I.e., to authenticate $M_1,\dots,M_n$, apply CBC-MAC to $(n,M_1,\dots,M_n)$.
  - Works since now message space is prefix-free.
  - Drawback: The message length (n) must be known in advance.
- "Solution 2": apply CBC-MAC to $(M_1,\dots,M_n,n)$
  - Message length does not have to be known is advance
  - But, this scheme is broken (see, M. Bellare, J. Kilian, P. Rogaway, The Security of Cipher Block Chaining, 1984)
- Solution 3: (preferable)
  - Use a second key K'.
  - Compute $MAC_{K,K'}(M_1,\dots,M_n) = E_{K'}(MAC_K(M_1,\dots,M_n))$
  - Essentially the same overhead as CBC-MAC

Hash functions

- MACs can be constructed based on hash functions.
- A hash function $h:X \rightarrow Y$ maps long inputs to fixed size outputs. ($|X|>|Y|$)
- No secret key. The hash function algorithm is public.
- If $|X|>|Y|$ there are collisions ($x \neq x'$ for which $h(x)=h(x')$), but it might be hard to find them.

Security definitions for hash functions

1. Weak collision resistance: for any $x \in X$, it is hard to find $x' \neq x$ such that $h(x)=h(x')$. (Also known as "universal one-way hash", or "*second* preimage resistance").
   - In other words, there is no efficient algorithm which is given *x* can find an *x'* such that $h(x)=h(x')$.

2. Strong collision resistance: it is hard to find any x,x' for which h(x)=h(x').
   - In other words, there is no no efficient algorithm which can find a pair *x,x'* such that $h(x)=h(x')$.

Security definitions for hash functions

- It is easier to find collisions.
  - In other words, under reasonable assumptions it holds that if it is possible to achieve security according to definition (2) then it is also possible to achieve security according to definition(1).
- Therefore strong collision resistance is a stronger assumption.

- Real world hash functions: MD5, SHA-1, SHA-256.

Hmm..

The Birthday Phenomenon (Paradox)

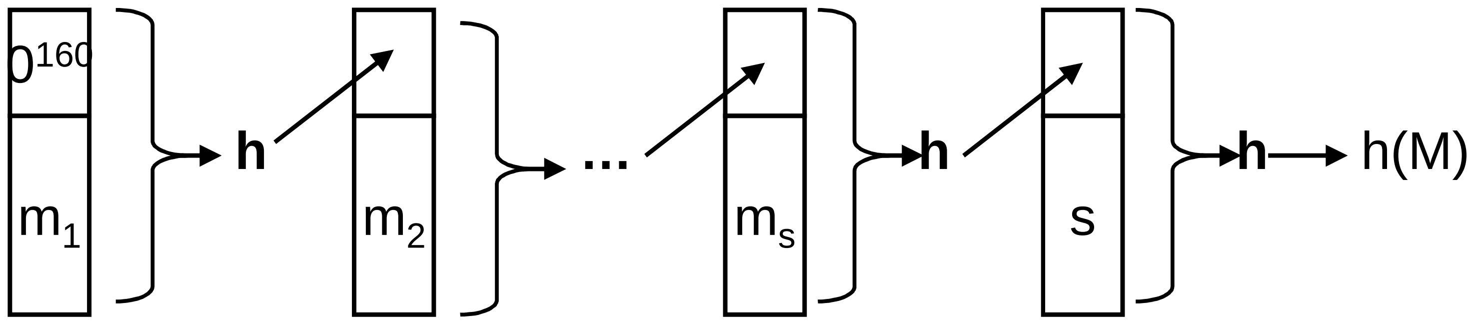
- For 23 people chosen at random, the probability that two of them have the same birthday is about ½.
- Compare to: The probability that one or more of them has the same birthday as Alan Turing is 23/365 (actually, $1-(1-1/365)^{23}$.)

- More generally, for a random $h: X \to Z$, if we choose about $|Z|^{½}$ elements of X at random ($1.17\ |Z|^{½}$), the probability that two of them are mapped to the same image is > ½.

- Implication: it's harder to achieve strong collision resistance
  - A random function with an n bit output
    - Can find x,x' with h(x)=h(x') after about $2^{n/2}$ tries.
    - Can find $x \neq 0$ s.t. h(x)=h(0) after about 2^n attempts.

From collision-resistance for fixed length inputs, to collision-resistance for arbitrary input lengths

- Hash function:
  - Input block length is usually 512 bits (|X|=512)
  - Output length is at least 160 bits (birthday attacks)
- Extending the domain to arbitrary inputs (Damgard-Merkle)
  - Suppose $h:\{0,1\}^{512} \rightarrow \{0,1\}^{160}$
  - Input: $M=m_1 \ldots m_s$, $|m_i|=512-160=352$. (what if $|M| \neq 352 \cdot i$ bits?)
  - Define: $y_0=0^{160}$. $y_i=h(y_{i-1},m_i)$. $y_{s+1}=h(y_s,s)$. $h(M)=y_{s+1}$.
  - Why is it secure? What about different length inputs?



Proof

- Show that if we can find M≠M' for which H(M)=H(M'), we can find blocks m ≠ m' for which h(m)=h(m').

- Case 1: suppose |M|=s, |M'|=s', and s ≠ s'
  - Then, collision: $H(M)=h(y_s,s) = h(y_{s'},s')=H(M')$
- Case 2: |M|=|M'|=s
  - We know that $H(M)=h(y_s,s)=h(y'_s,s)=H(M')$
  - If $y_s \neq y'_s$ then we found a collision in h.
  - Otherwise, go from i=s-1 to i=1:
    - $y_{i+1} = y'_{i+1}$ implies $h(y_i,m_{i+1}) = h(y'_I,m'_{i+1})$.
    - If $y_i \neq y'_i$ or $m_{i+1} \neq m'_{i+1}$, then we found a collision.
    - M ≠ M' and therefore there is an i for which $m_{i+1} \neq m'_{i+1}$

The implication of collisions

- Given a hash function with 2^n possible outputs. Collisions can be found
  - after a search of $2^{n/2}$ values
  - even faster if the function is weak (MD5, SHA-1)
- We find x, x' such that h(x)=h(x'), but we cannot control the value of x, x'.
- Can we find "meaningful" colliding values x, x' ?
  - The case of pdf files…

Basing MACs on Hash Functions

- Hash functions are not keyed. MAC_K uses a key.
- Best attack should not succeed with prob > $\max(2^{-|k|}, 2^{-|MAC()|})$.
- Idea: MAC combines message and a secret key, and hashes them with a collision resistant hash function.
  - E.g. $MAC_K(m) = h(k,m)$. (insecure.., given $MAC_K(m)$ can compute $MAC_K(m,|m|,m')$, if using the MD construction)
  - $MAC_K(m) = h(m,k)$. (insecure.., regardless of key length, use a birthday attack to find m,m' such that h(m)=h(m').)

- How should security be proved?:
  - Show that if MAC is insecure then so is hash function h.
  - Insecurity of MAC: adversary can generate $MAC_K(m)$ without knowing k.
  - Insecurity of h: adversary finds collisions ($x \neq x'$, $h(x)=h(x')$.)

HMAC

- Input: message *m,* a key *K*, and a hash function *h*.
- $HMAC_K(m) = h(K \oplus opad, h(K \oplus ipad, m))$
  - where ipad, opad are 64 byte long fixed strings
  - K is 64 byte long (if shorter, append 0s to get 64 bytes).
- Overhead: the same as that of applying h to m, plus an additional invocation to a short string.

- It was proven [BCK] that if HMAC is broken then either
  - h is not collision resistant (even when the initial block is random and secret), or
  - The output of h is not “unpredcitable” (when the initial block is random and secret)
- HMAC is used everywhere (SSL, IPSec).

Public Key Encryption

Classical symmetric ciphers

- Alice and Bob share a private key *k.*
- System is secure as long as *k* is secret.
- Major problem: generating and distributing *k.*

Diffie and Hellman: “New Directions in Cryptography”, 1976.

- “We stand today on the brink of a revolution in cryptography. The development of cheap digital hardware has freed it from the design limitations of mechanical computing…

  …such applications create a need for new types of cryptographic systems which minimize the necessity of secure key distribution…

  …theoretical developments in information theory and computer science show promise of providing provably secure cryptosystems, changing this ancient art into a science.”

Diffie-Hellman

- Came up with the idea of public key cryptography

Everyone can learn Bob's public key and encrypt messages to Bob.
Only Bob knows the decryption key and can decrypt.

Key distribution is greatly simplified.