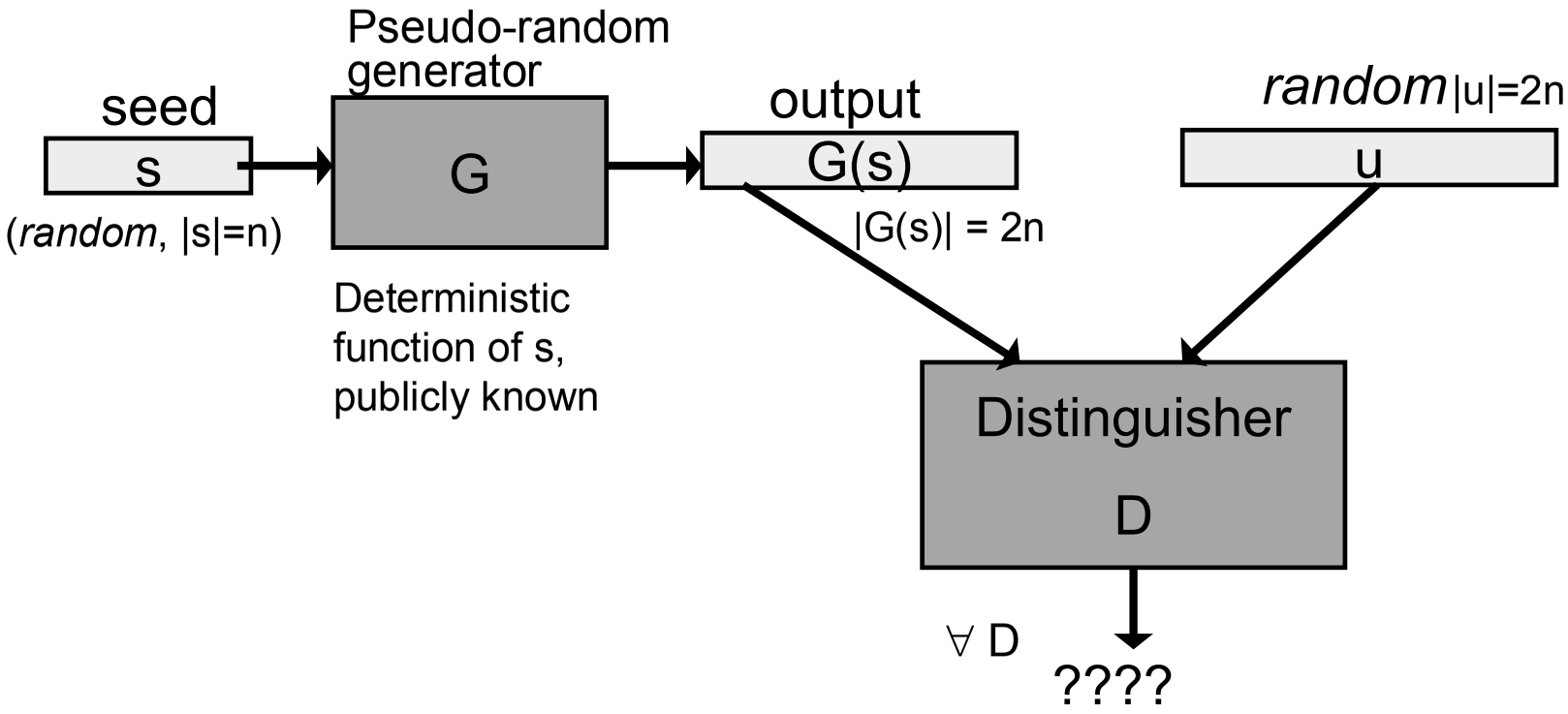
Introduction to Cryptography

Lecture 3

Benny Pinkas

Pseudo-random generator



Pseudo-random generators

- Pseudo-random generator (PRG)
  - $G: \{0,1\}^n \Rightarrow \{0,1\}^m$
    - A deterministic function, computable in polynomial time.
    - It must hold that m > n. Let us assume m=2n.
    - The function has only 2^n possible outputs.
- Pseudo-random property:
  - $\forall$ polynomial time adversary D, (whose output is 0/1)
    if we choose inputs $s \in_R \{0,1\}^n$, $u \in_R \{0,1\}^m$, (in other words, choose s and u uniformly at random), then
    it holds that D(G(s)) is similar to D(u)
    namely, | Pr[D(G(s))=1] - Pr[D(u)=1] | is negligible

P vs. NP

- If P=NP then PRGs do not exist (why?)
- So their existence can only be conjectured until the P=NP question is resolved.

Using a PRG for Encryption

- Replace the one-time-pad with the output of the PRG

- Key: a (short) random key $k \in \{0,1\}^{|k|}$.
- Message $m = m_1,\ldots,m_{|m|}$.
- Use a PRG $G : \{0,1\}^{|k|} \rightarrow \{0,1\}^{|m|}$
- Key generation: choose $k \in \{0,1\}^{|k|}$ uniformly at random.
- Encryption:
  - Use the output of the PRG as a one-time pad. Namely,
  - Generate $G(k) = g_1,\ldots,g_{|m|}$
  - Ciphertext $C = g_1 \oplus m_1,\ldots, g_{|m|} \oplus m_{|m|}$

- This is an example of a *stream cipher.*

Security of encryption against polynomial adversaries

- Perfect security (previous equivalent defs):
  - (indistinguishability) $\forall$ $m_0,m_1 \in M$, $\forall c$, the probability that c is an encryption of m_0 is equal to the probability that c is an encryption of m_1.
  - (semantic security) The distribution of m given the encryption of m is the same as the a-priori distribution of m.
- Security of pseudo-random encryption (equivalent defs):
  - (indistinguishability) $\forall$ $m_0,m_1 \in M$, no *polynomial time* adversary D can distinguish between the encryptions of m_0 and of m_1. Namely, $Pr[D(E(m_0))=1] \approx Pr[D(E(m_1))=1)$
  - (semantic security) $\forall$ $m_0,m_1 \in M$, a polynomial time adversary which is given $E(m_b)$, where $b \in_r \{0,1\}$, succeeds in finding b with probability $\approx$ ½.

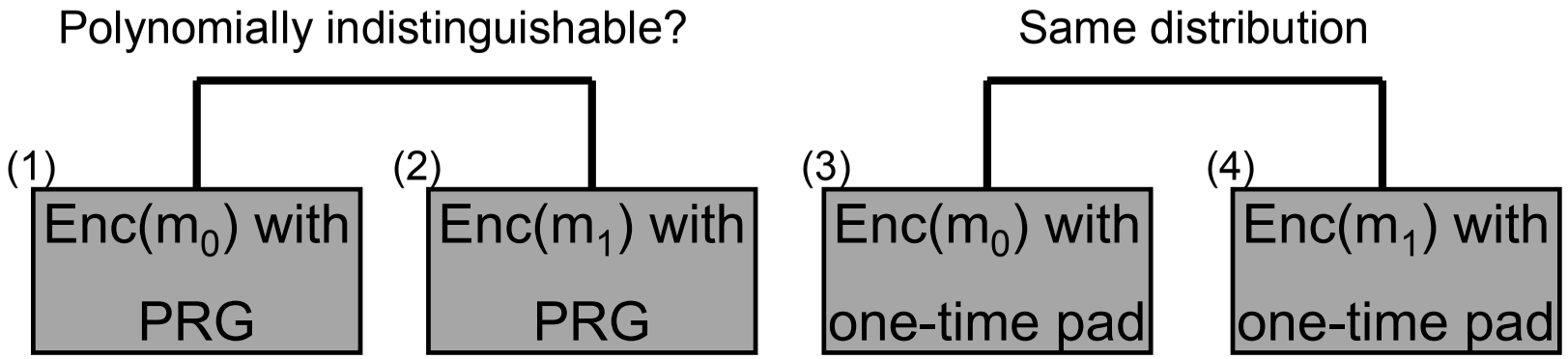
Proofs by reduction

- We don't know how to prove unconditional proofs of computational security; we must rely on assumptions.
  - We can simply assume that the encryption scheme is secure. This is bad.
  - Instead, we will assume that some low-level problem is hard to solve, and then prove that the cryptosystem is secure under this assumption.
  - (For example, the assumption might be that a certain function G is a pseudo-random generator.)
  - Advantages of this approach:
    - It is easier to design a low-level function.
    - There are (very few) "established" assumptions in cryptography, and people prove the security of cryptosystem based on these assumptions.

Using a PRG for Encryption: Security

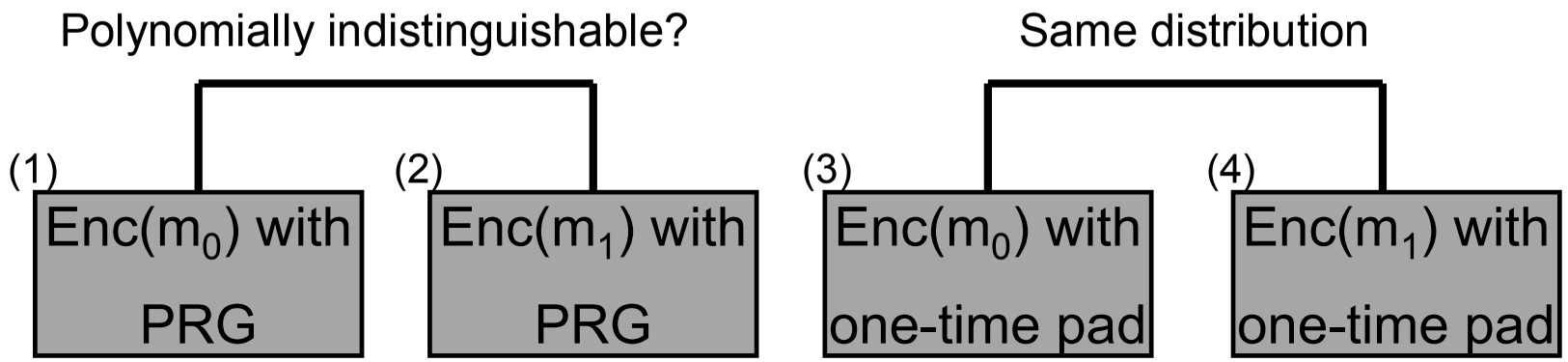
- The output of a pseudo-random generator is used instead of a one-time pad.
- Proof of security by reduction:
  - The assumption is that the PRG is strong (its output is indistinguishable from random).
  - We want to prove that in this case the encryption is strong (it satisfies the indistinguishability definition above).
  - In other words, prove that if one can break the security of the encryption (distinguish between encryptions of m_0 and of m_1), then it is also possible to break the security of the PRG (distinguish its output from random).

Proof of Security



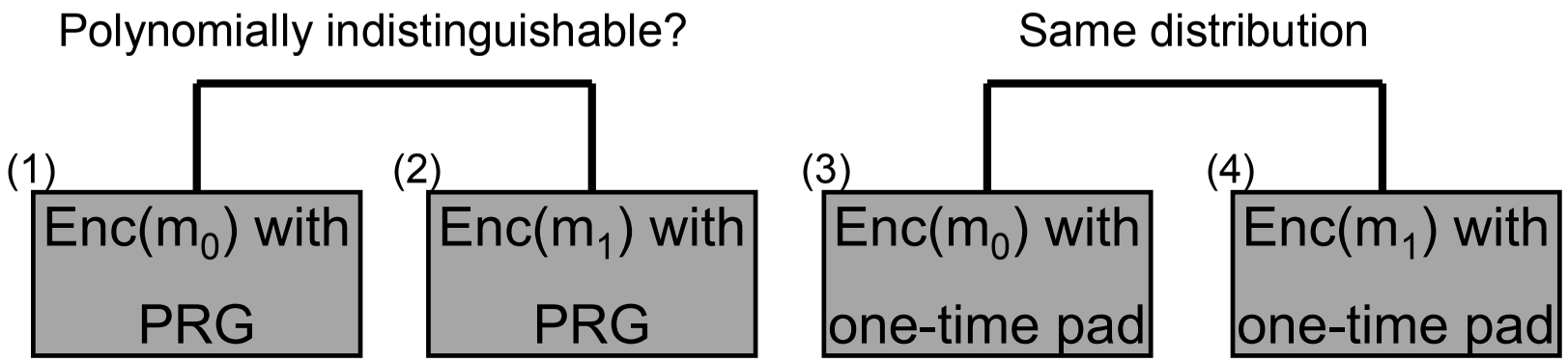
- Suppose that there is a D() which distinguishes between (1) and (2)
- We know that no D() can distinguish between (3) and (4)
- We are given a string S and need to decide whether it is drawn from a pseudorandom distribution or from a uniformly random distribution
- We will use S as a pad to encrypt a message.

Proof of Security



- Recall: we assume that there is a D() which always distinguishes between (1) and (2), and which distinguishes between (3) and (4) with probability ½.
- Choose a random $b \in \{0,1\}$ and compute $m_b \oplus S$. Give the result to D().
  - if S was chosen uniformly, D() must distinguish (3) from (4). (prob=½)
  - if S is pseudorandom, D() must distinguish (1) from (2). (prob=1)
- If D() outputs b then declare "pseudorandom", otherwise declare "random".
  - if S was chosen uniformly we output "pseudorandom" with prob ½.
  - if is pseudorandom we output "pseudorandom" with prob 1.

Proof of Security



- Recall: we assume that there is a D() which always distinguishes between (1) and (2), and which distinguishes between (3) and (4) with probability ½.
- Choose a random $b \in \{0,1\}$ and compute $m_b \oplus S$. Give the result to D().
  - if S was chosen uniformly, D() must distinguish (3) from (4). (prob=½)
  - if S is pseudorandom, D() must distinguish (1) from (2). (prob=½+δ)
- If D() outputs b then declare "pseudorandom", otherwise declare "random".
  - if S was chosen uniformly we output "pseudorandom" with prob ½.
  - if is pseudorandom we output "pseudorandom" with prob ½+δ.

Stream ciphers

- Stream ciphers are based on pseudo-random generators.
  - Usually used for encryption in the same way as OTP
- Examples: A5, SEAL, RC4.
  - Very fast implementations.
  - RC4 is popular and secure when used correctly, but it was shown that its first output bytes are biased. This resulted in breaking WEP encryption in 802.11.
- Some technical issues:
  - Stream ciphers require *synchronization* (for example, if some packets are lost in transit).

RC4

- Designed by Ron Rivest. Intellectual property belongs to RSA Inc.
  - Designed in 1987.
  - Kept secret until the design was leaked in 1994.
- Used in many protocols (SSL, etc.)
- Byte oriented operations.
- 8-16 machine operations per output byte.
- First output bytes are biased ☹

RC4 initialization

Word size is a single byte.

1. $j = 0$
2. $S_0 = 0;\ S_1 = 1;\ \ldots\ ;\ S_{255} = 255$
3. Let the key be $k_0;\ldots;k_{255}$ (repeating bits if key has fewer bits)
4. For $i = 0$ to 255
   - $j = (j + S_i + k_i) \bmod 256$
   - Swap S_i and S_j

RC4 keying stream generation

An output byte `B` is generated as follows:

- $i = i + 1 \bmod 256$
- $j = j + S_i \bmod 256$
- Swap S_i and S_j
- $r = S_i + S_j \bmod 256$
- $B = S_r$

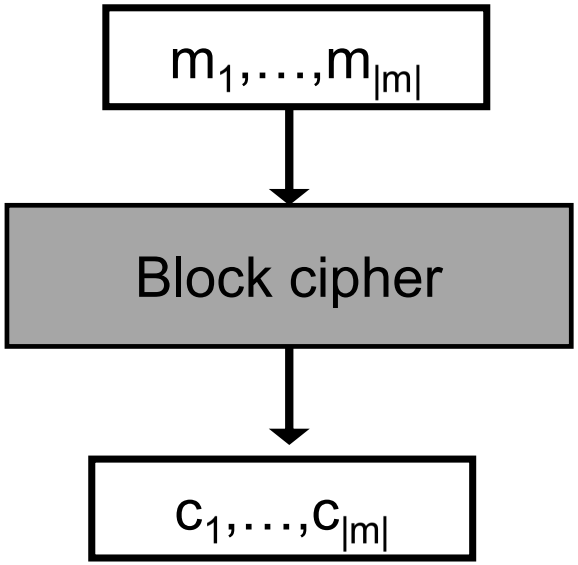
`B` is xored to the next byte of the plaintext.

Bias: The probability that the first two output bytes are 0 is $2^{-16}+2^{-23}$

Block Ciphers

- Plaintexts, ciphertexts of fixed length, |m|. Usually, |m|=64 or |m|=128 bits.
- The encryption algorithm E_k is a *permutation* over $\{0,1\}^{|m|}$, and the decryption D_k is its inverse. (They *are not* permutations of the bit order, but rather of the entire string.)

- Ideally, use a *random* permutation.
  - Can only be implemented using a table with $2^{|m|}$ entries ☹
- Instead, use a *pseudo-random* permutation*, keyed by a key k.
  - Implemented by a computer program whose input is m,k.
  - (*) will be explained shortly

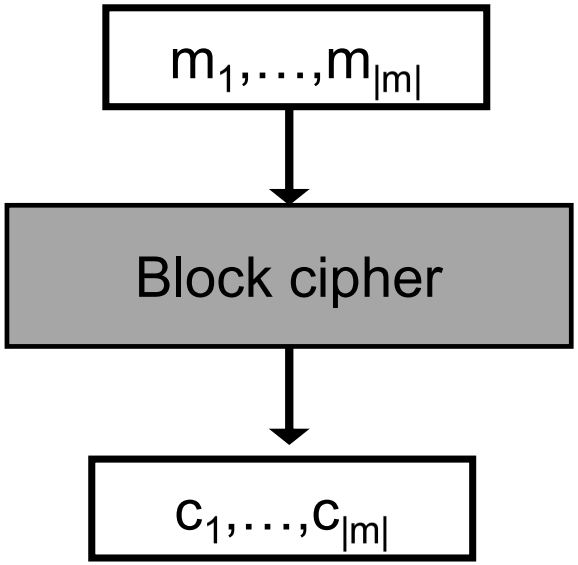


Block Ciphers

- Modeled as a pseudo-random permutation.

- Encrypt/decrypt whole blocks of bits
  - Might provide better encryption by simultaneously working on a block of bits
  - One error in ciphertext affects whole block
  - Delay in encryption/decryption
  - There was more research on the security of block ciphers than on the security of stream ciphers.

- Different *modes of operation* (for encrypting longer inputs)



Pseudo-random functions

- $F : \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^*$
  - The first input is the key, and once chosen it is kept fixed.
  - For simplicity, assume $F : \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}^n$
  - $F(k,x)$ is written as $F_k(x)$

- F is pseudo-random if $F_k()$ (where k is chosen uniformly at random) is indistinguishable (to a polynomial distinguisher D) from a function *f* chosen at random from all functions mapping $\{0,1\}^n$ to $\{0,1\}^n$
  - There are 2^n choices of F_k, whereas there are $(2^n)^{2^n}$ choices for *f*.
  - The distinguisher D's task:
    - We choose a function G. With probability ½ G is F_k (where $k \in_R \{0,1\}^n$), and with probability ½ it is a random function *f*.
    - D can compute $G(x_1),G(x_2),\ldots$ for any $x_1,x_2,\ldots$ it chooses.
    - D must say if $G=F_k$ or $G=f$.
    - F_k is pseudo-random if D succeeds with prob ½+negligible..

Pseudo-random permutations

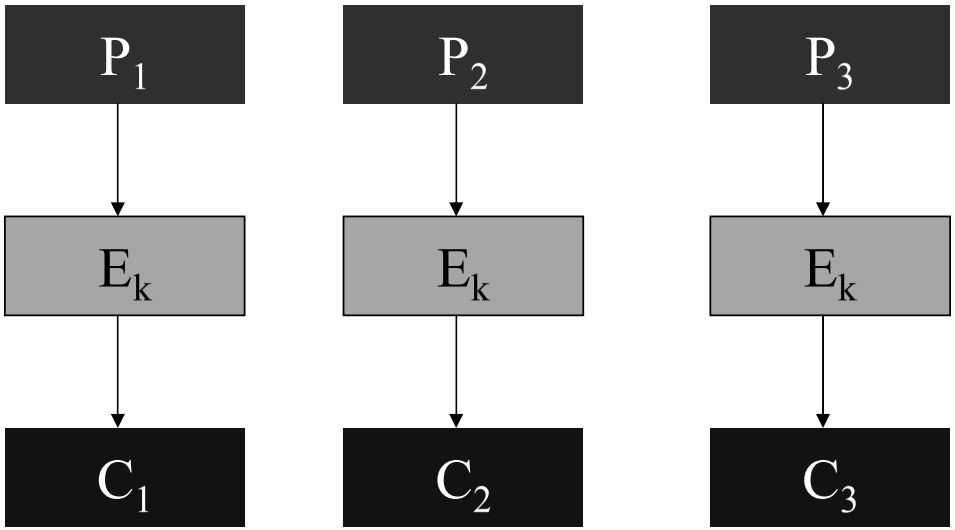
- $F_k(x)$ is a keyed permutation if for every choice of k, $F_k()$ is one-to-one.
  - Note that in this case $F_k(x)$ has an inverse, namely for every y there is exactly one x for which $F_k(x)=y$.

- $F_k(x)$ is a pseudo-random permutation if
  - It is a keyed permutation
  - It is indistinguishable (to a polynomial distinguisher D) from a permutation *f* chosen at random from all permutations mapping $\{0,1\}^n$ to $\{0,1\}^n$.
    - 2^n possible values for F_k
    - $(2^n)!$ possible values for a random permutation

Block ciphers

- A block cipher is a function $F_k(x)$ of a key k and an |m| bit input x, which has an |m| bit output.
  - $F_k(x)$ is a keyed permutation
- How can we encrypt plaintexts longer than |m|?
- Different modes of operation were designed for this task.

ECB Encryption Mode (Electronic Code Book)



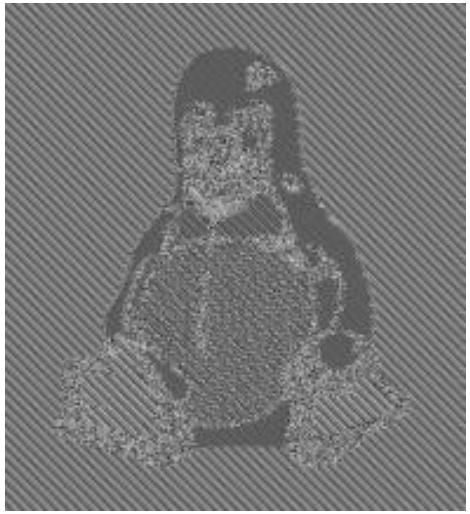
Namely, encrypt each plaintext block separately.

Properties of ECB

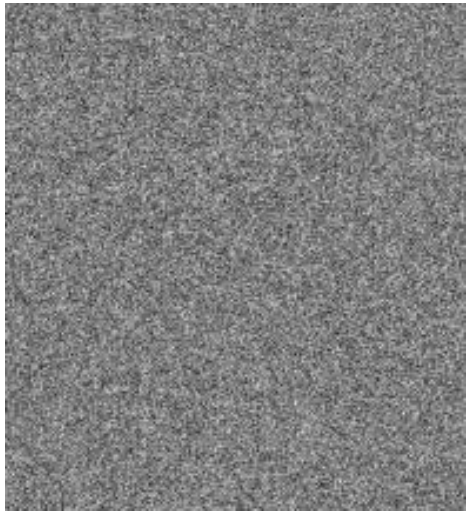
- Simple and efficient ☺
- Parallel implementation is possible ☺
- Does not conceal plaintext patterns ☹
  - Enc(P_1, P_2, P_1, P_3)
- Active attacks are easy ☹ (plaintext can be easily manipulated by removing, repeating, or interchanging blocks).

Encrypting bitmap images in ECB mode

original

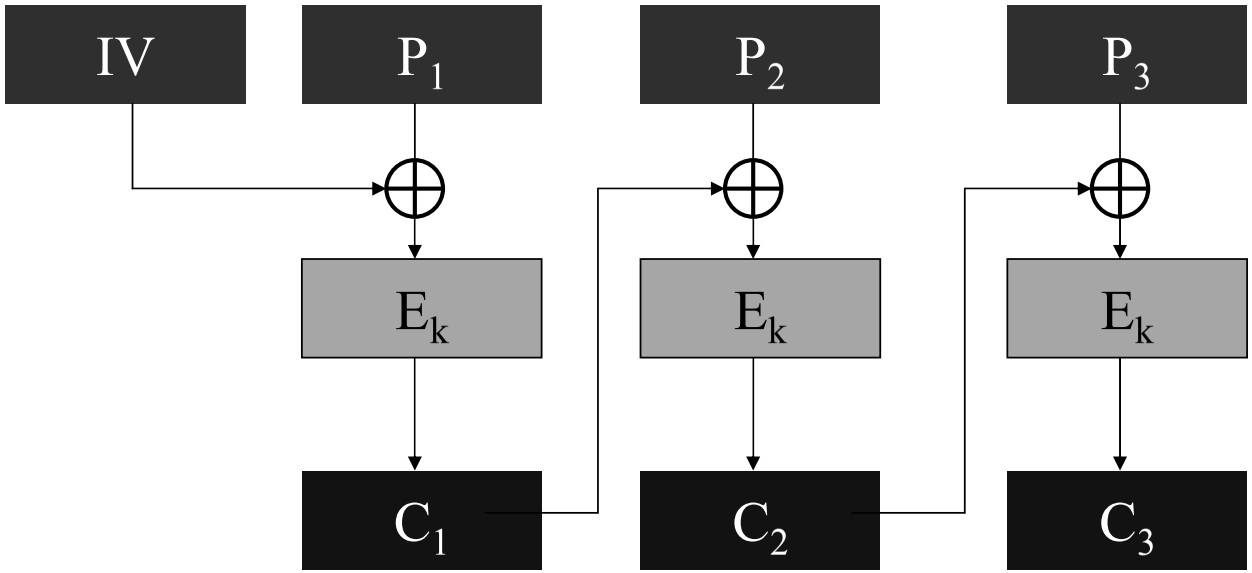


encrypted using ECB mode



encrypted using a secure mode

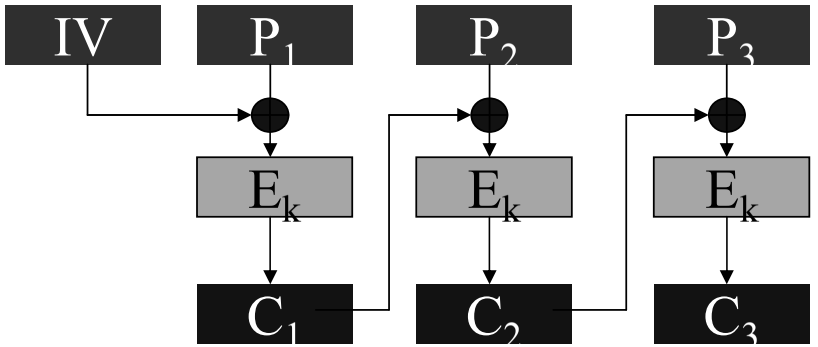
CBC Encryption Mode (Cipher Block Chaining)



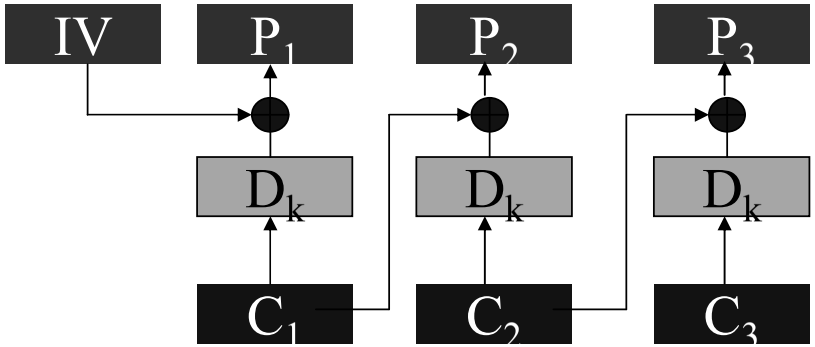
Previous *ciphertext* is XORed with current *plaintext* before encrypting current block.
An initialization vector IV is used as a "seed" for the process.
IV can be transmitted in the clear (unencrypted).

CBC Mode

Encryption:



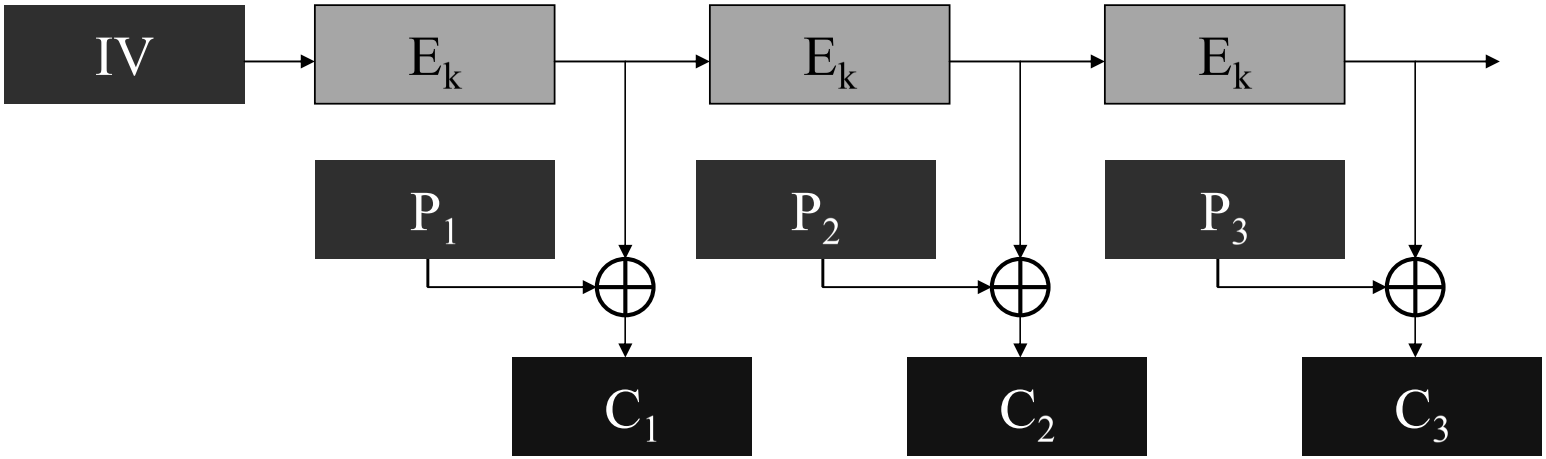
Decryption:



Properties of CBC

- Asynchronous: the receiver can start decrypting from any block in the ciphertext. ☺
- Errors in one *ciphertext* block propagate to the decryption of the next block (but that's it). ☺
- Conceals plaintext patterns (same block $\Rightarrow$ different ciphertext blocks) ☺
  - If IV is chosen at random, and E_K is a pseudo-random permutation, CBC provides chosen-plaintext security.
  - But if IV is fixed, CBC does not even hide not common *prefixes.*
- No parallel implementation is known ☹
- Plaintext cannot be easily manipulated ☺
- Standard in most systems: SSL, IPSec, etc.

OFB Mode (Output FeedBack)



- An initialization vector IV is used as a "seed" for generating a sequence of "pad" blocks
  - $E_k(IV)$, $E_k(E_k(IV))$, $E_k(E_k(E_k(IV)))$,…
- Essentially a stream cipher.
- IV can be sent in the clear. Must never be repeated.

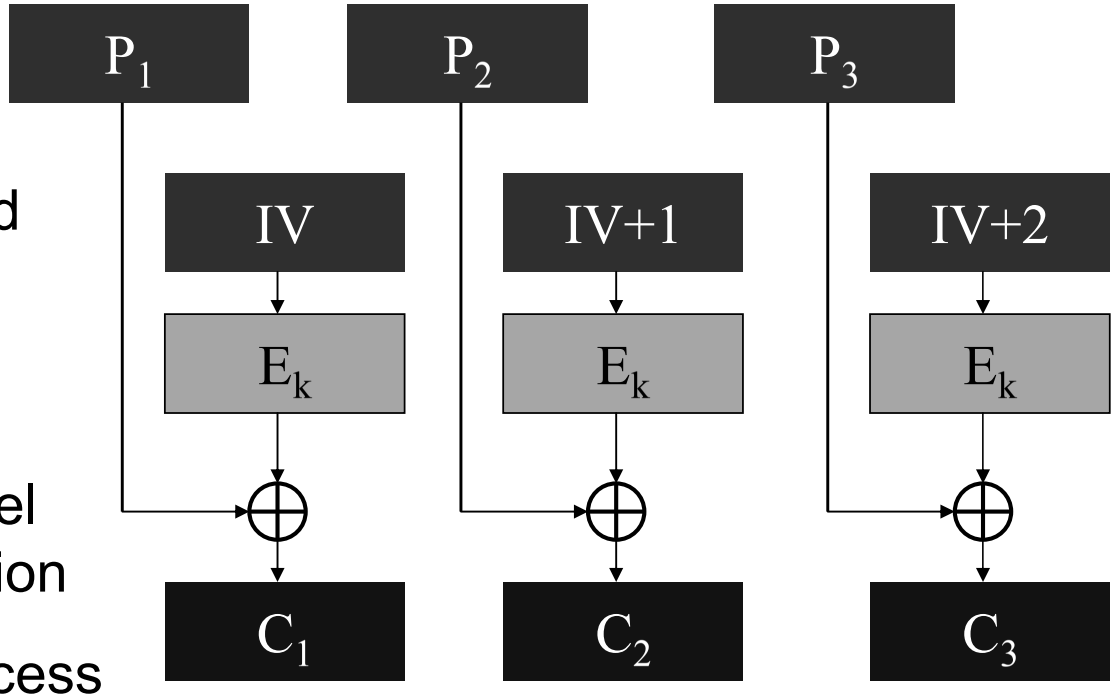
Properties of OFB

- Essentially implements a synchronous stream cipher. I.e., the two parties must know s_0 and the current bit position.
  - A block cipher can be used instead of a PRG.
  - The parties must synchronize the location they are encrypting/decrypting. ☹
- Conceals plaintext patterns. If IV is chosen at random, and E_K is a pseudo-random permutation, CBC provides chosen-plaintext security. ☺
- Errors in ciphertext do not propagate ☺
- Implementation:
  - Pre-processing is possible ☺
  - No parallel implementation is known ☹
- Active attacks (by manipulating the plaintext) are possible ☹

CTR (counter) Encryption Mode

IV is selected as a random value

- easy parallel implementation
- random access
- preprocessing



Design of Block Ciphers

- More an art/engineering challenge than science. Based on experience and public scrutiny.
  - *"Diffusion":* each intermediate/output bit affected by many input bits
  - *"Confusion"*: avoid structural relationships between bits
- Cascaded (round) design: the encryption algorithm is composed of iterative applications of a simple round

Confusion-Diffusion and Substitution-Permutation Networks

- Construct a PRP for a large block using PRPs for small blocks
- Divide the input to small parts, and apply rounds:
  - Feed the parts through PRPs *("confusion")*
  - Mix the parts *("diffusion")*
  - Repeat
- Why both confusion and diffusion are necessary?
- Design musts: Avalanche effect. Using reversible s-boxes.

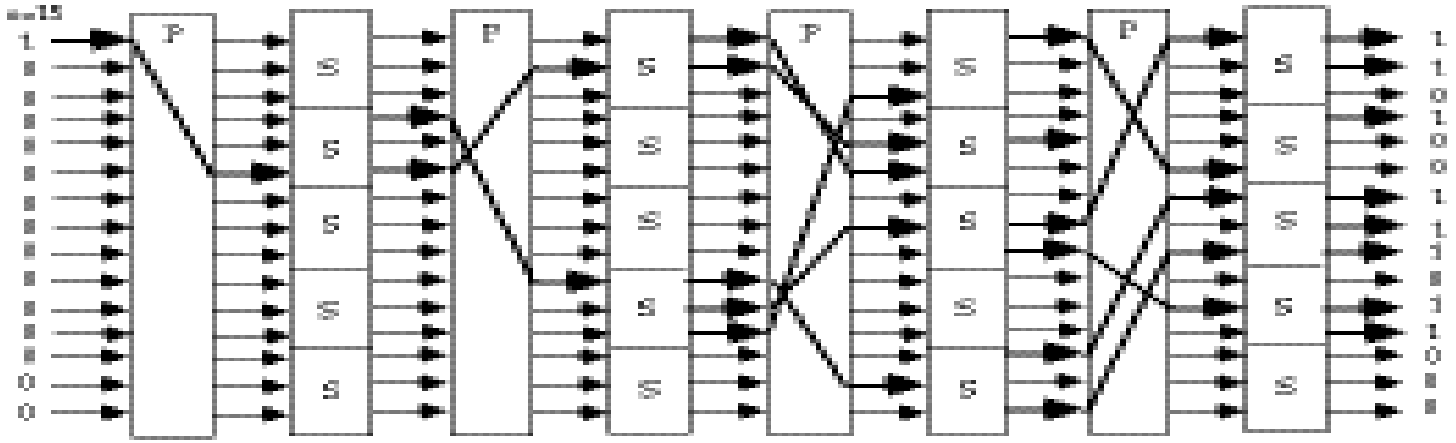


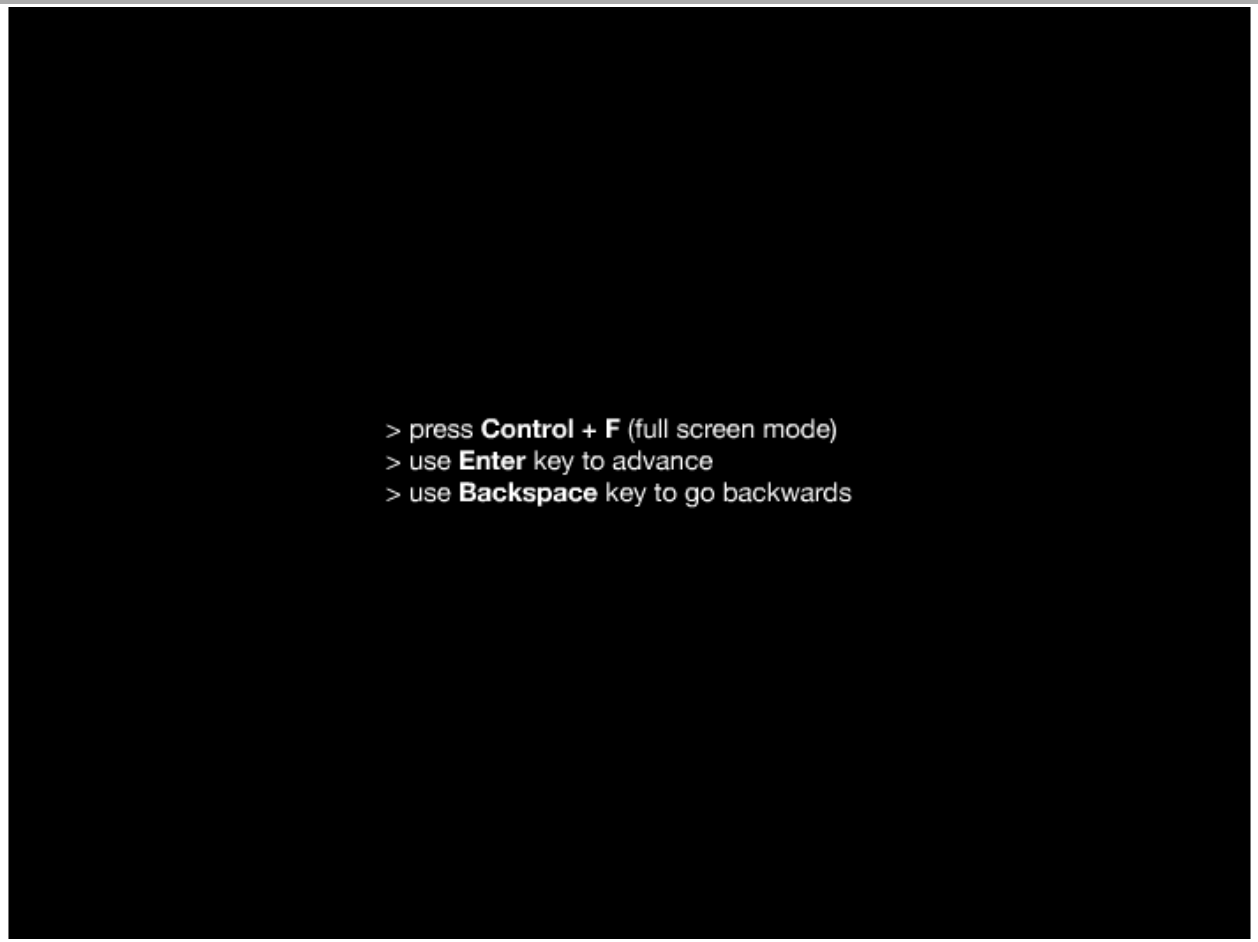
Fig 2.3 - Substitution-Permutation Network with the Avalanche Characteristic

AES (Advanced Encryption Standard)

- Design initiated in 1997 by NIST
  - Goals: improve security and software efficiency of DES
  - 15 submissions, several rounds of public analysis
  - The winning algorithm: Rijndael

- Input block length: 128 bits
- Key length: 128, 192 or 256 bits
- Multiple rounds (10, 12 or 14), but does not use a Feistel network

Rijndael animation



Reversible s-boxes

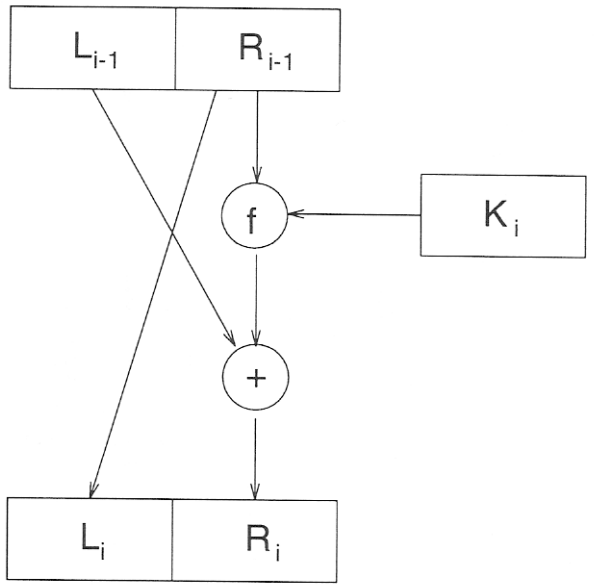
- Substitution-Permutation networks must use reversible s-boxes
  - Allow for easy decryption
- However, we want the block cipher to be “as random as possible”
  - s-boxes need to have some structure to be reversible
  - Better use non-invertible s-boxes

- Enter Feistel networks
  - A round-based block-cipher which uses s-boxes which are not necessarily reversible
  - Namely, building an invertible function (permutation) from a non-invertible function.

Feistel Networks

- Encryption:
- *Input:* $P = L_{i-1} \mid R_{i-1}$. $|L_{i-1}|=|R_{i-1}|$
  - $L_i = R_{i-1}$
  - $R_i = L_{i-1} \oplus F(K_i, R_{i-1})$
- Decryption?

- No matter which function is used as F, we obtain a permutation (i.e., F is reversible even if *f* is not).
- The same code/circuit, with keys in reverse order, can be used for decryption.
- Theoretical result [LubRac]: If *f* is a pseudo-random *function* then a 4 rounds Feistel network gives a pseudo-random *permutation*



DES (Data Encryption Standard)

- A Feistel network encryption algorithm:
  - How many rounds?
  - How are the round keys generated?
  - What is F?

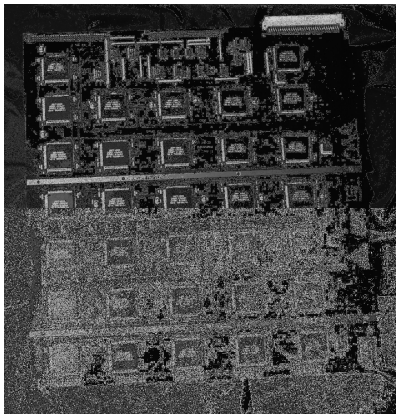
- DES (Data Encryption Standard)
  - Designed by IBM and the NSA, 1977.
  - 64 bit input and output
  - 56 bit key
  - 16 round Feistel network
  - Each round key is a 48 bit subset of the key
- Throughput ≈ software: 10Mb/sec, hardware: 1Gb/sec (in 1991!).

Security of DES

- Criticized for unpublished design *decisions* (designers did not want to disclose differential cryptanalysis).
- Very secure – the best attack in practice is brute force
  - 2006: $1 million search machine: 30 seconds
    - cost per key: less than $1
  - •2006: 1000 PCs at night: 1 month
    - Cost per key: essentially 0 (+ some patience)
- Some theoretical attacks were discovered in the 90s:
  - Differential cryptanalysis
  - Linear cryptanalysis: requires about 2^{40} known plaintexts
- The use of DES is not recommend since 2004 , but 3-DES is still recommended for use.

Double DES

- DES is out of date due to brute force attacks on its short key (56 bits)



- Why not apply DES twice with two keys?
  - Double DES: $DES_{k1,k2} = E_{k2}(E_{k1}(m))$
  - Key length: 112 bits
- But, double DES is susceptible to a meet-in-the-middle attack, requiring $\approx 2^{56}$ operations and storage.
  - Compared to brute a force attack, requiring 2^{112} operations and O(1) storage.

Meet-in-the-middle attack

- Meet-in-the-middle attack
  - $c = E_{k2}(E_{k1}(m))$
  - $D_{k2}(c) = E_{k1}(m)$

- The attack:
  - Input: *(m,c)* for which $c = E_{k2}(E_{k1}(m))$
  - For every possible value of k_1, generate and store $E_{k1}(m)$.
  - For every possible value of k_2, generate and store $D_{k2}(c)$.
  - Match k_1 and k_2 for which $E_{k1}(m) = D_{k2}(c)$.
  - Might obtain several options for (k_1,k_2). Check them or repeat the process again with a new *(m,c)* pair (see next slide)

- The attack is applicable to any iterated cipher. Running time and memory are $O(2^{|k|})$, where |k| is the key size.

Meet-in-the-middle attack: how many pairs to check?

- The plaintext and the ciphertext are 64 bits long
- The key is 56 bits long

- Suppose that we are given one plaintext-ciphertext pair (m,c)
  - The attack looks for k1,k2, such that $D_{k2}(c) = E_{k1}(m)$
  - The correct values of k1,k2 satisfies this equality
  - There are 2^{112} (actually $2^{112}-1$) other values for k_1,k_2.
  - Each one of these satisfies the equalities with probability 2^{-64}
  - We therefore expect to have $2^{112-64}=2^{48}$ candidates for k_1,k_2.

- Suppose that we are given one pairs (m,c), (m',c')
  - The correct values of k1,k2 satisfies both equalities
  - There are 2^{112} (actually $2^{112}-1$) other values for k_1,k_2.
  - Each one of these satisfies the equalities with probability 2^{-128}
  - We therefore expect to have $2^{112-128}<1$ false candidates for k_1,k_2.

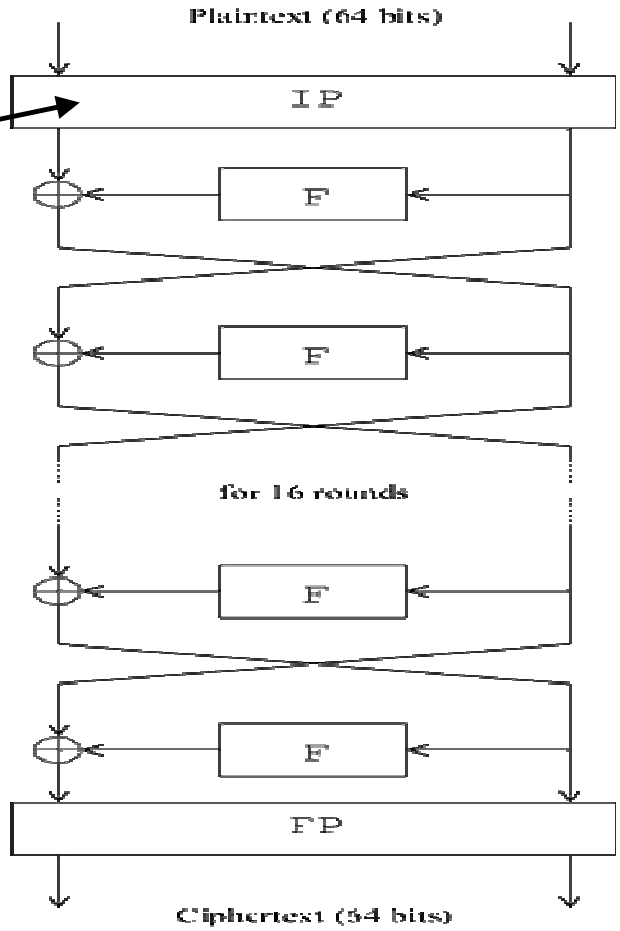
Triple DES

- 3DES $_{k1,k2} = E_{k1}(D_{k2}(E_{k1}(m))$
- Why use Enc(Dec(Enc( ))) ?
  - Backward compatibility: setting $k_1=k_2$ is compatible with single key DES
- Only two keys
  - Effective key length is 112 bits
  - Why not use three keys? There is a meet-in-the-middle attack with 2^{112} operations
- 3DES provides good security. Widely used. Less efficient.

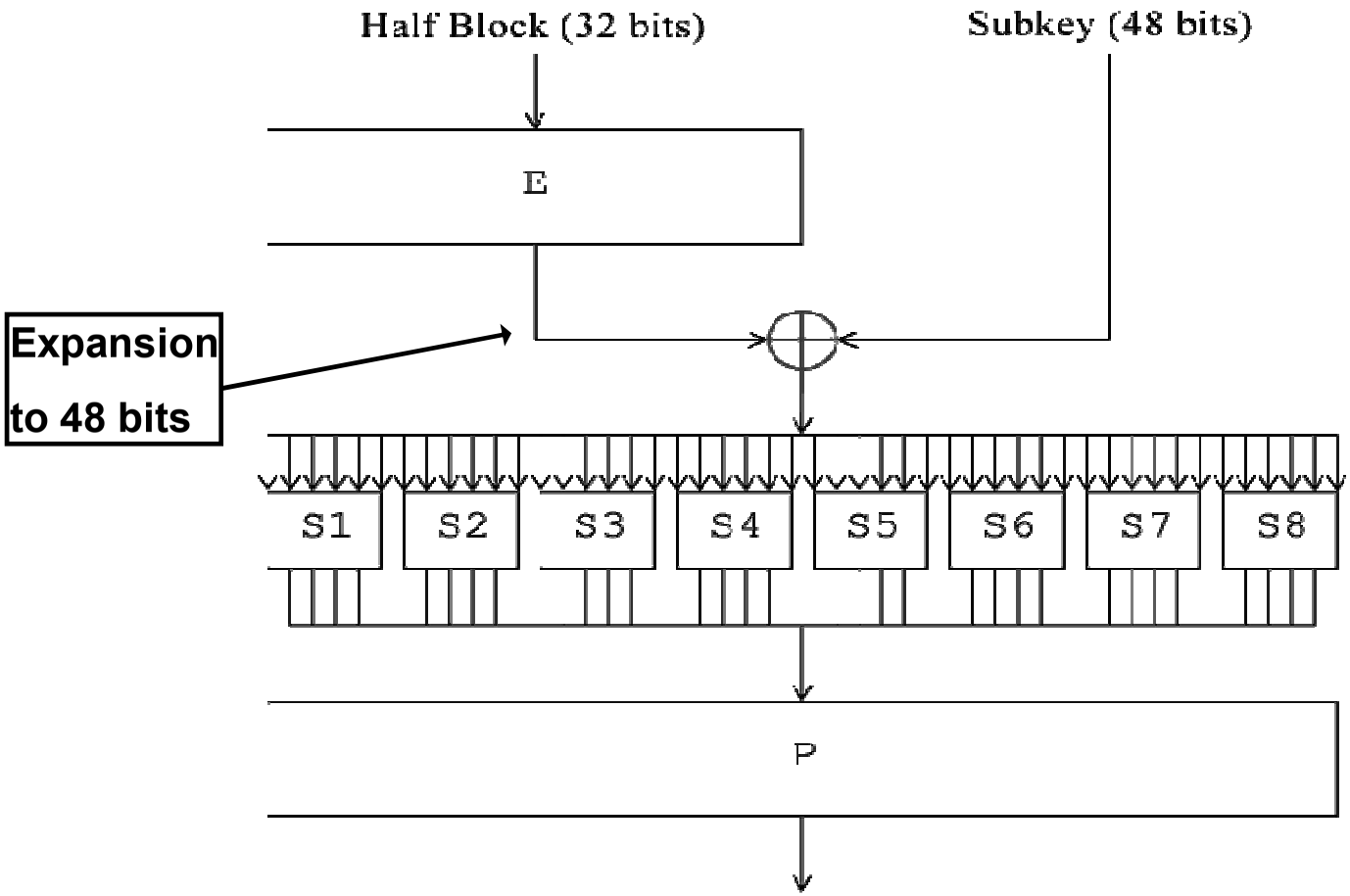
Attacking DES

Initial permutation of bit locations:

- not secret
- makes implementations in software less efficient



DES F functions



The S-boxes

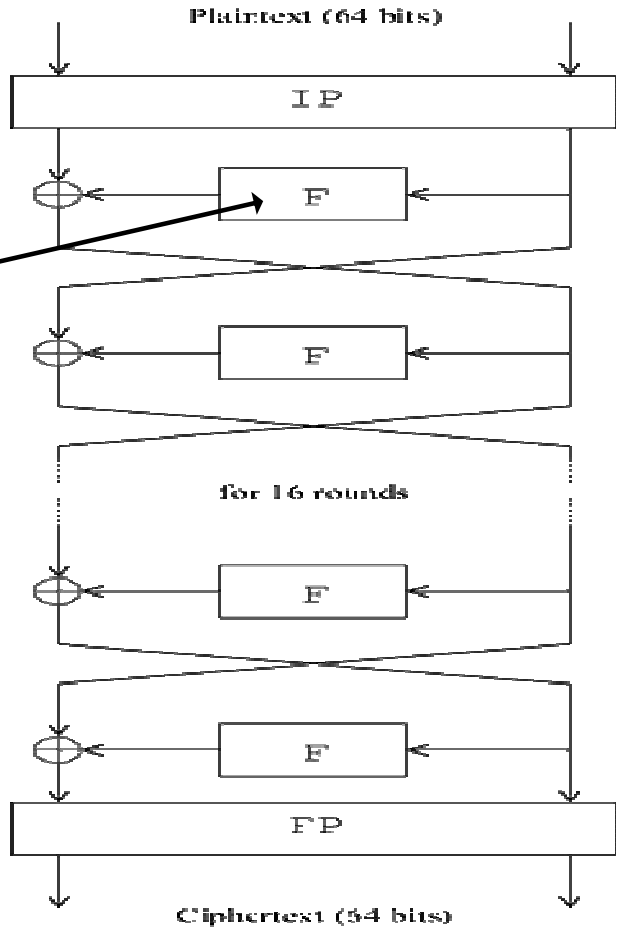
- Very careful design (it is now known that random choices for the S-boxes result in weak encryption).

- Each s-box maps 6 bits to 4 bits:
  - A 4×16 table of 4-bit entries.
  - Bits 1 and 6 choose the row, and bits 2-5 choose column.
  - Each row is a *permutation* of the values 0,1,…,15.
    - Therefore, given an output there are exactly 4 options for the input
  - Changing one input bit changes at least two output bits $\Rightarrow$ avalanche effect.

Differential Cryptanalysis of DES

DES diagram:

S-boxes



Differential Cryptanalysis [Biham-Shamir 1990]

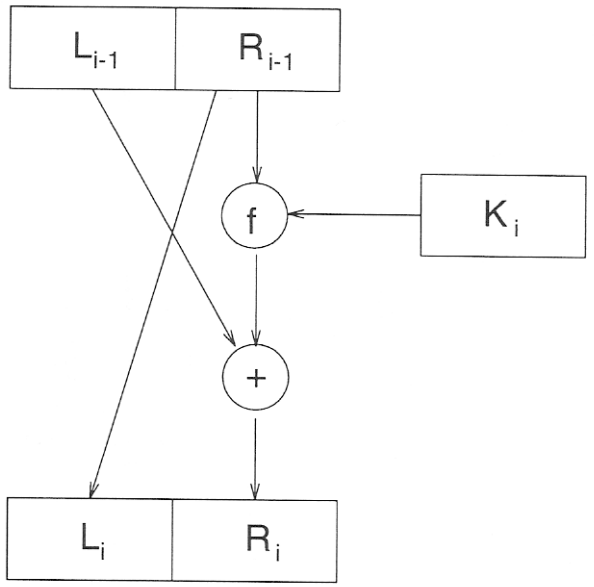
- The first attack to reduce the overhead of breaking DES to below exhaustive search
- Very powerful when applied to other encryption algorithms

- Depends on the structure of the encryption algorithm
- Observation: all operations except for the s-boxes are linear
- Linear operations:
  - $a = b \oplus c$
  - a = the bits of b in (known) permuted order
- Linear relations can be exposed by solving a system of linear equations

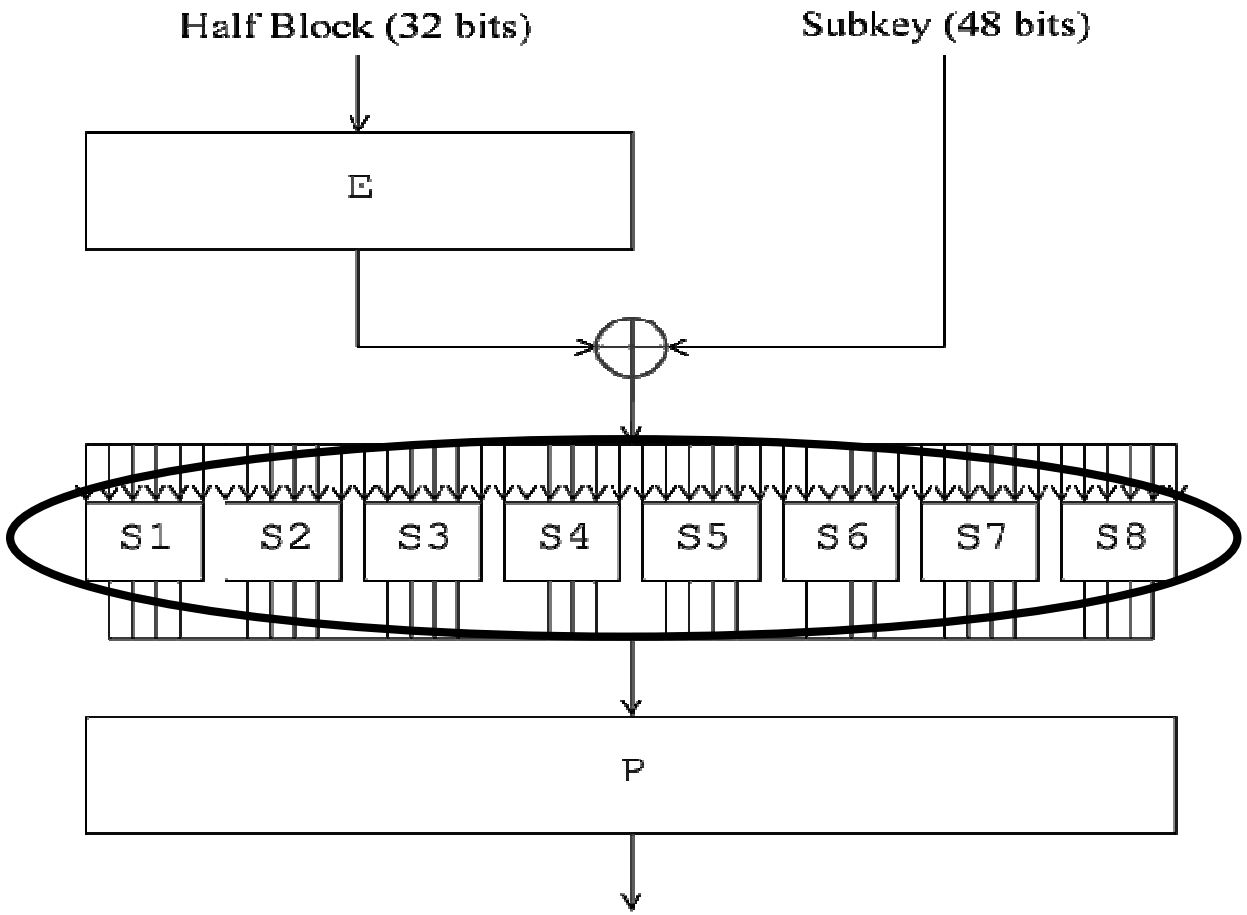
A Linear F in a Feistel Network?

- Suppose $F(R_{i-1},K_i) = R_{i-1} \oplus K_i$
  - Namely, that F is linear
- Then $R_i = L_{i-1} \oplus R_{i-1} \oplus K_i$

  $L_i = R_{i-1}$
- Write L_{16}, R_{16} as linear functions of L_0, R_0 and K.
  - Given L_0R_0 and $L_{16}R_{16}$ Solve and find K.
- F must therefore be non-linear.
- F is the only source of non-linearity in DES.



DES F functions



Source of non-linearity

Differential Cryptanalysis

- The S-boxes are non-linear
- We study the differences between two encryptions of two different plaintexts

- Notation:
  - The plaintexts are P and P*
  - Their difference is $dP = P \oplus P^*$
  - Let X and X* be two intermediate values, for P and P*, respectively, in the encryption process.
  - Their difference is $dX = X \oplus X^*$
    - Namely, dX is always the result of two inputs

Differences and S-boxes

- S-box: a function (table) from 6 bit inputs to 4 bit output
- X and X* are inputs to the same S-box. We can compute their difference $dX = X \oplus X^*$.
- $Y = S(X)$
- When $dX=0$, $X=X^*$, and therefore $Y=S(X)=S(X^*)=Y^*$, and $dY=0$.
- When $dX\neq 0$, $X\neq X^*$ and we don't know dY for sure, but we can investigate its distribution.
- For example,

Distribution of Y' for S1

- dX=110100
- There are 2^6=64 input pairs with this difference, { (000000,110100), (000001,110101),…}
- For each pair we can compute the xor of outputs of S1
- E.g., S1(000000)=1110, S1(110100)=1001. dY=0111.
- Table of frequencies of each dY:

0000	0001	0010	0011	0100	0101	0110	0111
0	8	16	6	2	0	0	12
1000	1001	1010	1011	1100	1101	1110	1111
6	0	0	0	0	8	0	6

Differential Probabilities

- The probability of $dX \Rightarrow dY$ is the probability that a pair of inputs whose xor is dX, results in a pair of outputs whose xor is dY (for a given S-box).
- Namely, for dX=110100 these are the entries in the table divided by 64.

- Differential cryptanalysis uses entries with large values
  - $dX=0 \Rightarrow dY=0$
  - Entries with value 16/64
  - (Recall that the outputs of the S-box are uniformly distributed, so the attacker gains a lot by looking at differentials rather than the original values.)