Introduction to Cryptography

Lecture 2

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Perfect Cipher

- What type of security would we like to achieve?
- "Given C, the adversary has no idea what M is"
  - Impossible since adversary might have a-priori information
- In an "ideal" world, the message will be delivered in a magical way, out of the reach of the adversary
  - We would like to achieve similar security
- Definition: a *perfect cipher*
  - *Pr(plaintext = P* **|** *ciphertext = C) = Pr(plaintext = P)*

Perfect Ciphers

- A simple criteria for perfect ciphers.
- Claim: The cipher is perfect if, and only if,

  $\forall$ $m_1, m_2 \in M$, $\forall$cipher c,

  $Pr(Enc(m_1)=c) = Pr(Enc(m_2)=c)$. (homework)
- Idea: Regardless of the plaintext, the adversary sees the same distribution of ciphertexts.
- Note that the proof cannot assume that the cipher is the one-time-pad, but rather only that *Pr(plaintext = P | ciphertext = C) = Pr(plaintext = P)*

Size of key space

- Theorem: For a perfect encryption scheme, the number of keys is at least the size of the message space.
- Proof:
  - Consider ciphertext C.
  - Must be a possible encryption of any plaintext m.
  - But, need a different key per message m.
- Corollary: Key length of one-time pad is optimal ☹

Computational security

- We should only worry about polynomial adversaries
- Idea: Generate a string which "looks random" to any polynomial adversary. Use it instead of a OTP.
- Looks random?
  - Fraction of bits set to 1 is ≈ 50%
  - Longest run of 0's is of length ≈ log(n),
  - Is that sufficient?...
- Enumerating a set of statistical tests that the string should pass is not enough.

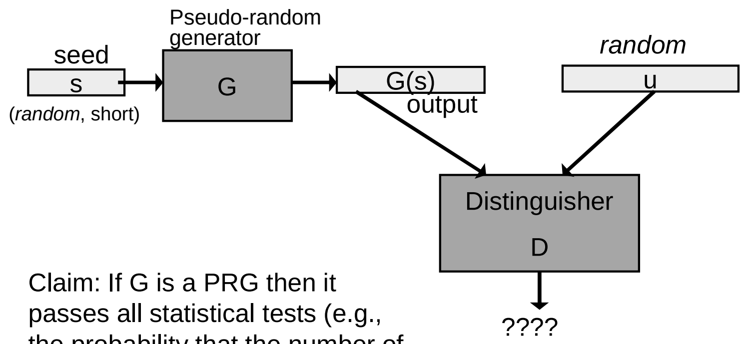
Computational security – Pseudo-randomness

- Pseudo-random string: no *efficient* observer can *distinguish* it from a uniformly random string of the same length
- Motivation: *Indistinguishable objects are equivalent*
- The foundation of modern cryptography
- *(t,ε)*-Pseudo-random generator (PRG)
  - G: $\{0,1\}^{|k|} \Rightarrow \{0,1\}^{|m|}$ $|k|<|m|$, polynomially computable.
  - $\forall$ adversary D running in time *t*,
    for $s \in_R \{0,1\}^{|k|}$, $u \in_R \{0,1\}^{|m|}$,
    it holds that $\Pr(D(G(s)) \neq D(u)) < \varepsilon$

Pseudo-random generators

- Pseudo-random generator (PRG)
  - G: $\{0,1\}^{|k|} \Rightarrow \{0,1\}^{|m|}$ $|k|<|m|$, polynomially computable.
  - $\forall$ <u>polynomial time</u> adversary D,
    for $s \in_R \{0,1\}^{|k|}$, $u \in_R \{0,1\}^{|m|}$,
    it holds that $\Pr(D(G(s)) \neq D(u))$ is <u>negligible</u>
  - *Polynomial time:* running in time $t(|k|)$ s.t. $\exists$polynomial $p()$ for which $t(|k|)<p(|k|)$ for all large enough $|k|$
  - *Negligible:* the difference is a function $\varepsilon(|k|)$ s.t. $\forall$polynomials $q()$, for all large enough $|k|$ it holds that $\varepsilon(|k|) < 1/q(|k|)$

Pseudo-random generator



Claim: If G is a PRG then it passes all statistical tests (e.g., the probability that the number of 1 bits is < |m|/3 is negligible).

Using a PRG for Encryption

- Key: a (short) random seed $s \in \{0,1\}^{|k|}$.
- Message $m = m_1, \ldots, m_{|m|}$.

- Encryption:
  - Use the output of the PRG as a one-time pad. Namely,
  - Generate $G(s) = g_1, \ldots, g_{|m|}$
  - Ciphertext $C = g_1 \oplus m_1, \ldots, g_{|m|} \oplus m_{|m|}$

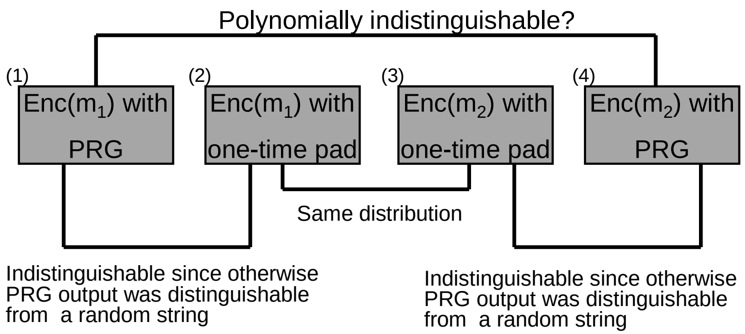
Using a PRG for Encryption: Security

- One time pad:
  - $\forall$ $m_1, m_2 \in M$, $\forall c$, the probability that c is an encryption of m_1 is equal to the probability that c is an encryption of m_2.
  - I.e., $\forall$ $m_1, m_2 \in M$ $\forall c$, it is impossible to tell whether c is an encryption of m_1 or of m_2.

- Security of pseudo-random encryption:
  - Show that $\forall$ $m_1, m_2 \in M$, no *polynomial time* adversary can distinguish between the encryptions of m_1 and of m_2.

- Proof by reduction: if one can break the security of the encryption (distinguish between encryptions of m_1 and of m_2), it can also break the security of the PRG (distinguish it from random).

Proof of Security



Distinguishing between (1) and (4), implies distinguishing between (1) and (2), or (2) and (3), or (3) and (4).

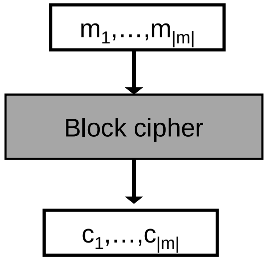
Symmetric systems used in practice

- Are not based on computational problems
- Are (usually) not proven secure by reductions
- Are designed for specific input and key lengths
- Are very efficient

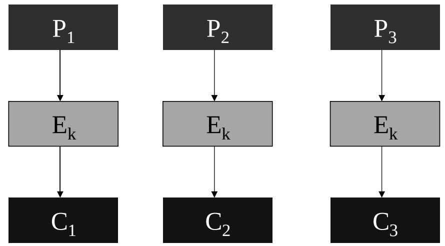
- Stream ciphers
  - Meant to implement a pseudo-random generator
  - Usually used for encryption in the same way as OTP
  - Examples: A5, RC4, SEAL.
  - Require *synchronization*

Block Ciphers

- Plaintexts, ciphertexts of fixed length, |m|. Usually, |m|=64 or |m|=128 bits.
- The encryption algorithm E_k is a *permutation* over $\{0,1\}^{|m|}$, and the decryption D_k is its inverse.
- Ideally, use a *random* permutation. Instead, use a *pseudo-random* permutation, keyed by a key k.
- Encrypt/decrypt whole blocks of bits
  - Might provide better encryption by simultaneously working on a block of bits
  - Error propagation: one error in ciphertext affects whole block
  - Delay in encryption/decryption
- Different modes of operation (for encrypting longer inputs)



ECB Encryption Mode (Electronic Code Book)

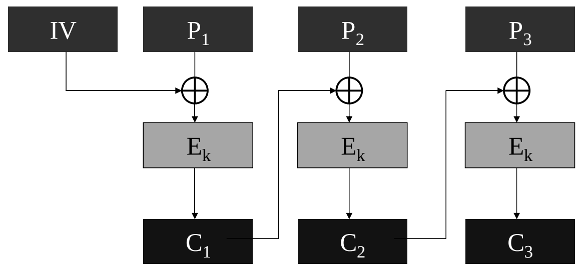


Namely, encrypt each plaintext block separately.

Properties of ECB

- Simple and efficient ☺
- Parallel implementation is possible ☺
- Does not conceal plaintext patterns ☹
  - Enc(P_1, P_2, P_1, P_3)
- Active attacks are possible ☹ (plaintext can be easily manipulated by removing, repeating, or interchanging blocks).

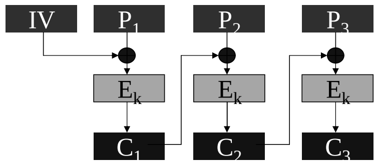
CBC Encryption Mode (Cipher Block Chaining)



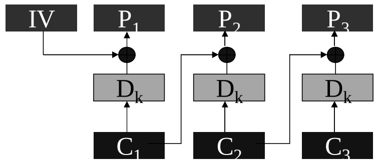
Previous *ciphertext* is XORed with current *plaintext* before encrypting current block.
An initialization vector IV is used as a “seed” for the process.
IV can be transmitted in the clear (unencrypted).

CBC Mode

Encryption:



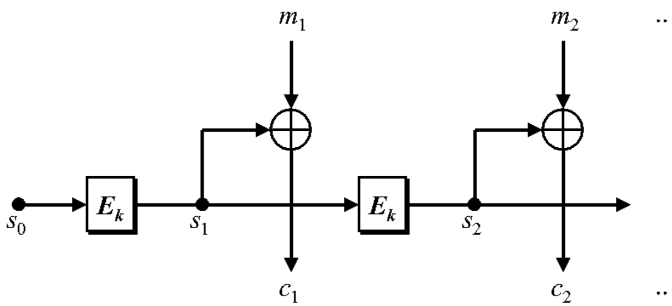
Decryption:



Properties of CBC

- Asynchronous: the receiver can start decrypting from any block in the ciphertext. ☺
- Errors in one *ciphertext* block propagate to the decryption of the next block (but that's it). ☺
- Conceals plaintext patterns (same block ⇒ different ciphertext blocks) ☺
  - But if IV is fixed, CBC does not hide not common *prefixes*
- No parallel implementation is known ☹
- Plaintext cannot be easily manipulated ☺
- Standard in most systems: SSL, IPSec, etc.

OFB Mode (Output FeedBack)



- An initialization vector s_0 is used as a "seed" for generating a sequence of "pad" blocks s_i. ($s_i=E_k(s_{i-1})$)
- Essentially a stream cipher
- s_0 can be sent in the clear.

Properties of OFB

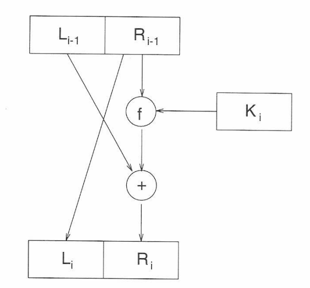
- Synchronous stream cipher. I.e., the two parties must know s_0 and the current bit position. ☹
- The parties must synchronize the location they are encrypting/decrypting. ☹
- Errors in ciphertext do not propagate ☺
- Implementation:
  - Pre-processing is possible ☺
  - No parallel implementation known ☹
- Conceals plaintext patterns ☺
- Active attacks (by manipulating the plaintext) are possible ☹

Design of Block Ciphers

- More an art/engineering challenge than science. Based on experience and public scrutiny.
- *"Diffusion":* each intermediate/output bit affected by many input bits
- *"Confusion"*: avoid structural relationships between bits
- Cascaded (round) design: the encryption algorithm is composed of iterative applications of a simple round
- A common round function: Feistel network

Feistel Networks

- Encryption:
- *Input:* P = L_{i-1} | R_{i-1}. $|L_{i-1}|=|R_{i-1}|$
  - $L_i = R_{i-1}$
  - $R_i = L_{i-1} \oplus F(K_i, R_{i-1})$
- Decryption?
- No matter which function is used as F, we obtain a permutation (i.e., F is reversible even if *f* is not).
- The same code/circuit, with keys in reverse order, can be used for decryption.
- Theoretical result [LubRac]: If F is a pseudo-random function then 4 rounds give a pseudo-random permutation



DES (Data Encryption Standard)

- A Feistel network encryption algorithm:
  - How many rounds?
  - How are the round keys generated?
  - What is F?
- DES (Data Encryption Standard)
  - Designed by IBM and the NSA, 1977.
  - 64 bit input and output
  - 56 bit key
  - 16 round Feistel network
  - Each round key is a 48 bit subset of the key
- Throughput ≈ software: 10Mb/sec, hardware: 1Gb/sec (in 1991!).
- Criticized for unpublished design *decisions* (designers did not want to disclose differential cryptanalysis).
- Linear cryptanalysis: about 2^{40} known plaintexts

What we've learned today

- Perfect security implies |M| ≤ |K|
- Computational security
- Pseudo-randomness, Pseudo-random generator
- Block ciphers
- DES