Introduction to Cryptography

Lecture 2

Benny Pinkas

Perfect Cipher

- What type of security would we like to achieve?
- “Given C, the adversary has no idea what M is”
  - Impossible since adversary might have a-priori information

- In an “ideal” world, the message will be delivered in a magical way, out of the reach of the adversary
  - We would like to achieve similar security

- Definition: a *perfect cipher*
  - *Pr(plaintext = P | ciphertext = C) = Pr(plaintext = P)*

Perfect Ciphers

- A simple criteria for perfect ciphers.
- Claim: The cipher is perfect if, and only if,

  $\forall\ m_1, m_2 \in M,\ \forall$cipher c,

  $Pr(Enc(m_1)=c) = Pr(Enc(m_2)=c).$ (homework)

- Idea: Regardless of the plaintext, the adversary sees the same distribution of ciphertexts.

- Note that the proof cannot assume that the cipher is the one-time-pad, but rather only that *Pr(plaintext = P | ciphertext = C) = Pr(plaintext = P)*

Size of key space

- Theorem: For a perfect encryption scheme, the number of keys is at least the size of the message space.

- Proof:
  - Consider ciphertext C.
  - Must be a possible encryption of any plaintext m.
  - But, need a different key per message m.

- Corollary: Key length of one-time pad is optimal ☹

Computational security

- We should only worry about polynomial adversaries
- Idea: Generate a string which “looks random” to any polynomial adversary. Use it instead of a OTP.

- Looks random?
  - Fraction of bits set to 1 is ≈ 50%
  - Longest run of 0’s is of length ≈ log(n),
  - Is that sufficient?...

- Enumerating a set of statistical tests that the string should pass is not enough.

Computational security – Pseudo-randomness

- Pseudo-random string: no *efficient* observer can *distinguish* it from a uniformly random string of the same length
- Motivation: *Indistinguishable objects are equivalent*
- The foundation of modern cryptography

- *(t,ε)*-Pseudo-random generator (PRG)
  - G: $\{0,1\}^{|k|} \Rightarrow \{0,1\}^{|m|}$ $|k|<|m|$, polynomially computable.
  - $\forall$ adversary D running in time *t*,
    for $s \in_R \{0,1\}^{|k|}$, $u \in_R \{0,1\}^{|m|}$,
    it holds that $\Pr(D(G(s)) \neq D(u)) < \varepsilon$

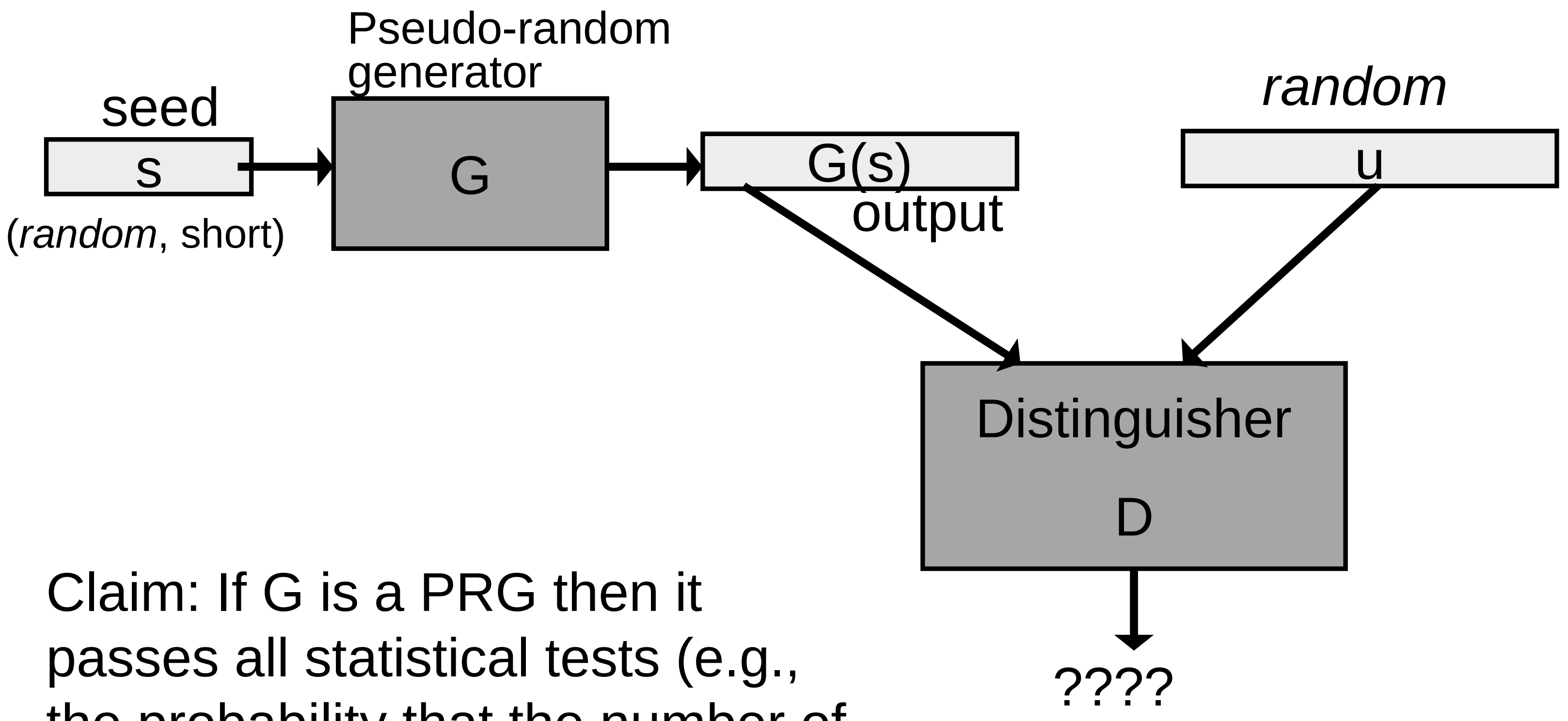
Pseudo-random generators

- Pseudo-random generator (PRG)
  - G: $\{0,1\}^{|k|} \Rightarrow \{0,1\}^{|m|}$ $|k|<|m|$,   polynomially computable.
  - $\forall$ polynomial time adversary D,
    for $s \in_R \{0,1\}^{|k|}$, $u \in_R \{0,1\}^{|m|}$,
    it holds that Pr(D(G(s)) ≠ D(u) is negligible

  - *Polynomial time:* running in time $t(|k|)$ s.t. $\exists$polynomial $p()$ for which $t(|k|)<p(|k|)$ for all large enough $|k|$

  - *Negligible:* the difference is a function $\varepsilon(|k|)$ s.t. $\forall$polynomials $q()$, for all large enough $|k|$ it holds that $\varepsilon(|k|) < 1/q(|k|)$

Pseudo-random generator



Claim: If G is a PRG then it passes all statistical tests (e.g., the probability that the number of 1 bits is < |m|/3 is negligible).

Using a PRG for Encryption

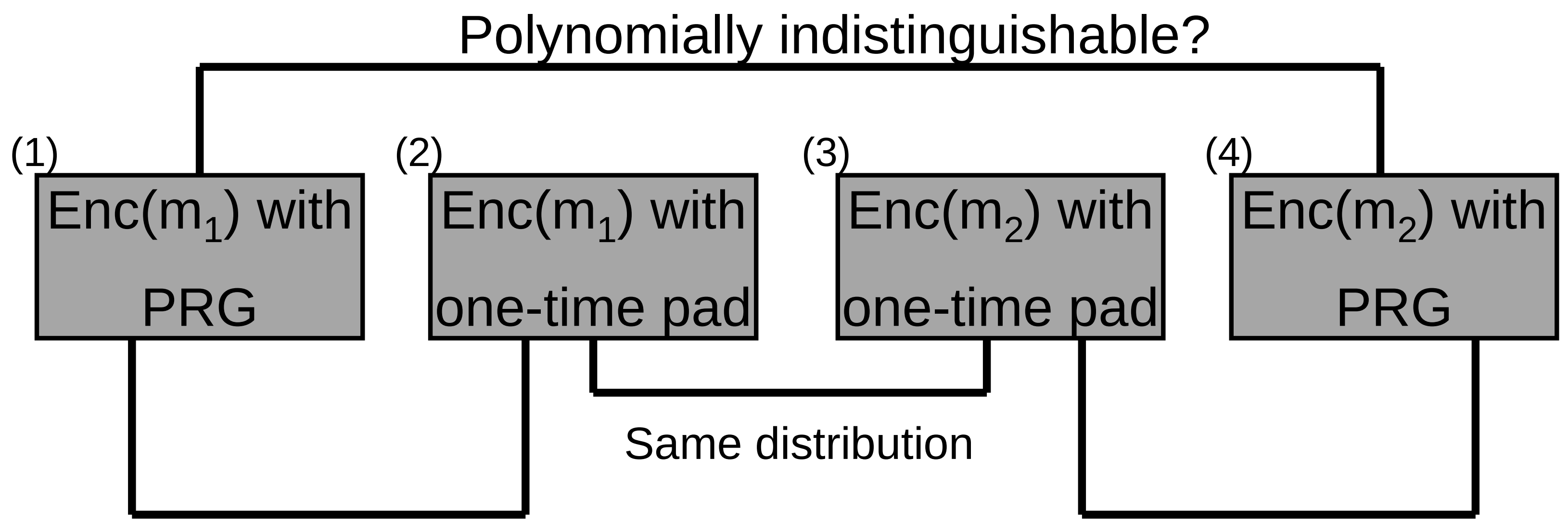
- Key: a (short) random seed $s \in \{0,1\}^{|k|}$.
- Message $m = m_1,\ldots,m_{|m|}$.

- Encryption:
  - Use the output of the PRG as a one-time pad. Namely,
  - Generate $G(s) = g_1,\ldots,g_{|m|}$
  - Ciphertext $C = g_1 \oplus m_1,\ldots, g_{|m|} \oplus m_{|m|}$

Using a PRG for Encryption: Security

- One time pad:
  - $\forall$ $m_1, m_2 \in M$, $\forall c$, the probability that c is an encryption of m_1 is equal to the probability that c is an encryption of m_2.
  - I.e., $\forall$ $m_1, m_2 \in M$ $\forall c$, it is impossible to tell whether c is an encryption of m_1 or of m_2.
- Security of pseudo-random encryption:
  - Show that $\forall$ $m_1, m_2 \in M$, no *polynomial time* adversary can distinguish between the encryptions of m_1 and of m_2.
- Proof by reduction: if one can break the security of the encryption (distinguish between encryptions of m_1 and of m_2), it can also break the security of the PRG (distinguish it from random).

Proof of Security


Polynomially indistinguishable?
(1)
$Enc(m_1)$ with PRG
(2)
$Enc(m_1)$ with one-time pad
(3)
$Enc(m_2)$ with one-time pad
(4)
$Enc(m_2)$ with PRG
Same distribution

Indistinguishable since otherwise PRG output was distinguishable from a random string

Indistinguishable since otherwise PRG output was distinguishable from a random string

Distinguishing between (1) and (4), implies distinguishing between (1) and (2), or (2) and (3), or (3) and (4).

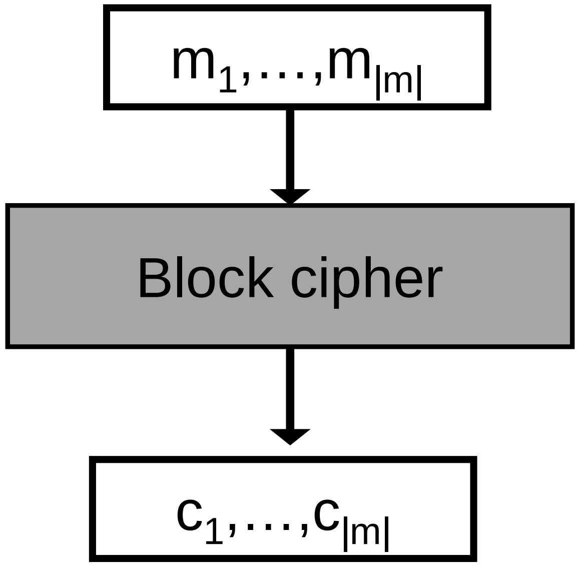
Symmetric systems used in practice

- Are not based on computational problems
- Are (usually) not proven secure by reductions
- Are designed for specific input and key lengths
- Are very efficient

- Stream ciphers
  - Meant to implement a pseudo-random generator
  - Usually used for encryption in the same way as OTP
  - Examples: A5, RC4, SEAL.
  - Require *synchronization*

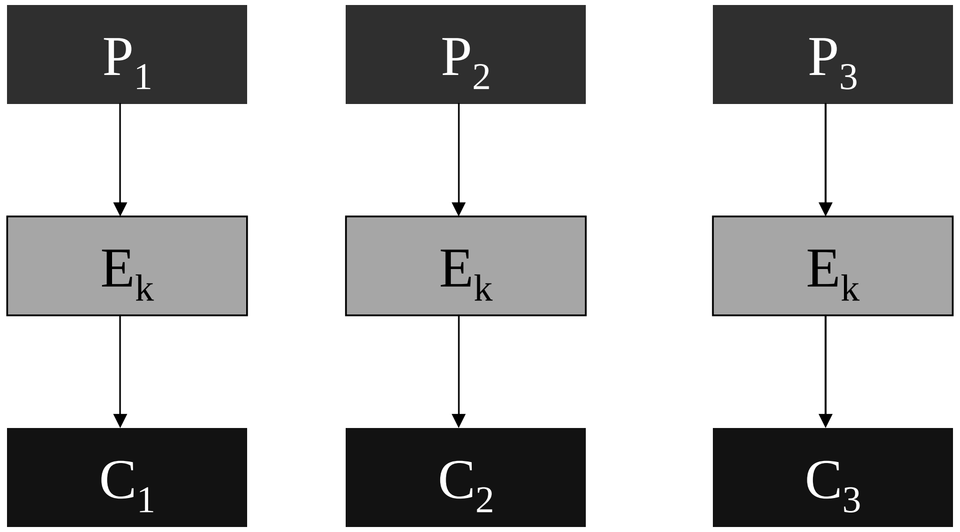
Block Ciphers

- Plaintexts, ciphertexts of fixed length, |m|. Usually, |m|=64 or |m|=128 bits.
- The encryption algorithm E_k is a *permutation* over $\{0,1\}^{|m|}$, and the decryption D_k is its inverse.
- Ideally, use a *random* permutation. Instead, use a *pseudo-random* permutation, keyed by a key k.



- Encrypt/decrypt whole blocks of bits
  - Might provide better encryption by simultaneously working on a block of bits
  - Error propagation: one error in ciphertext affects whole block
  - Delay in encryption/decryption
- Different modes of operation (for encrypting longer inputs)

ECB Encryption Mode (Electronic Code Book)

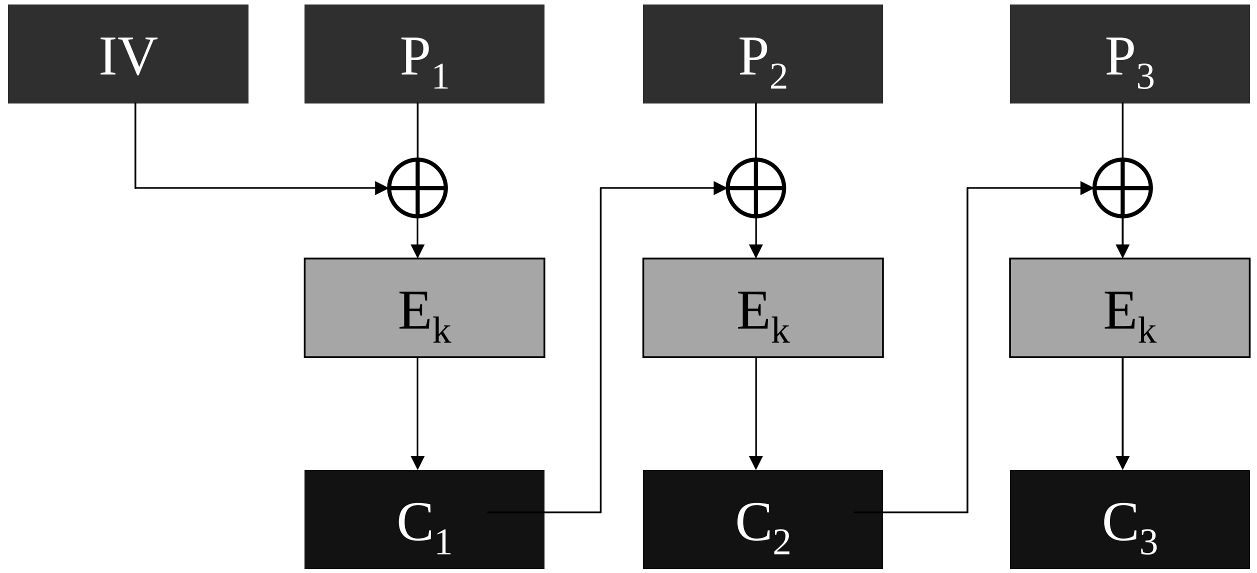


Namely, encrypt each plaintext block separately.

Properties of ECB

- Simple and efficient ☺
- Parallel implementation is possible ☺
- Does not conceal plaintext patterns ☹
  - Enc(P_1, P_2, P_1, P_3)
- Active attacks are possible ☹ (plaintext can be easily manipulated by removing, repeating, or interchanging blocks).

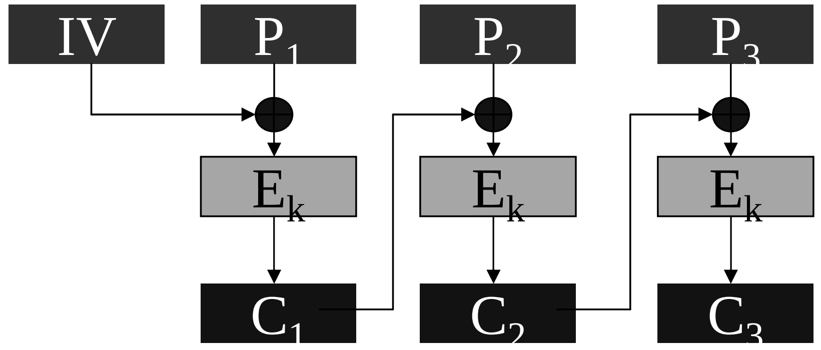
CBC Encryption Mode (Cipher Block Chaining)



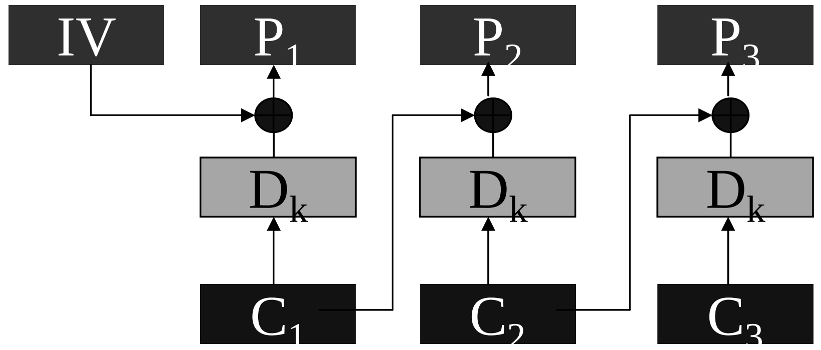
Previous *ciphertext* is XORed with current *plaintext* before encrypting current block.
An initialization vector IV is used as a “seed” for the process.
IV can be transmitted in the clear (unencrypted).

CBC Mode

Encryption:



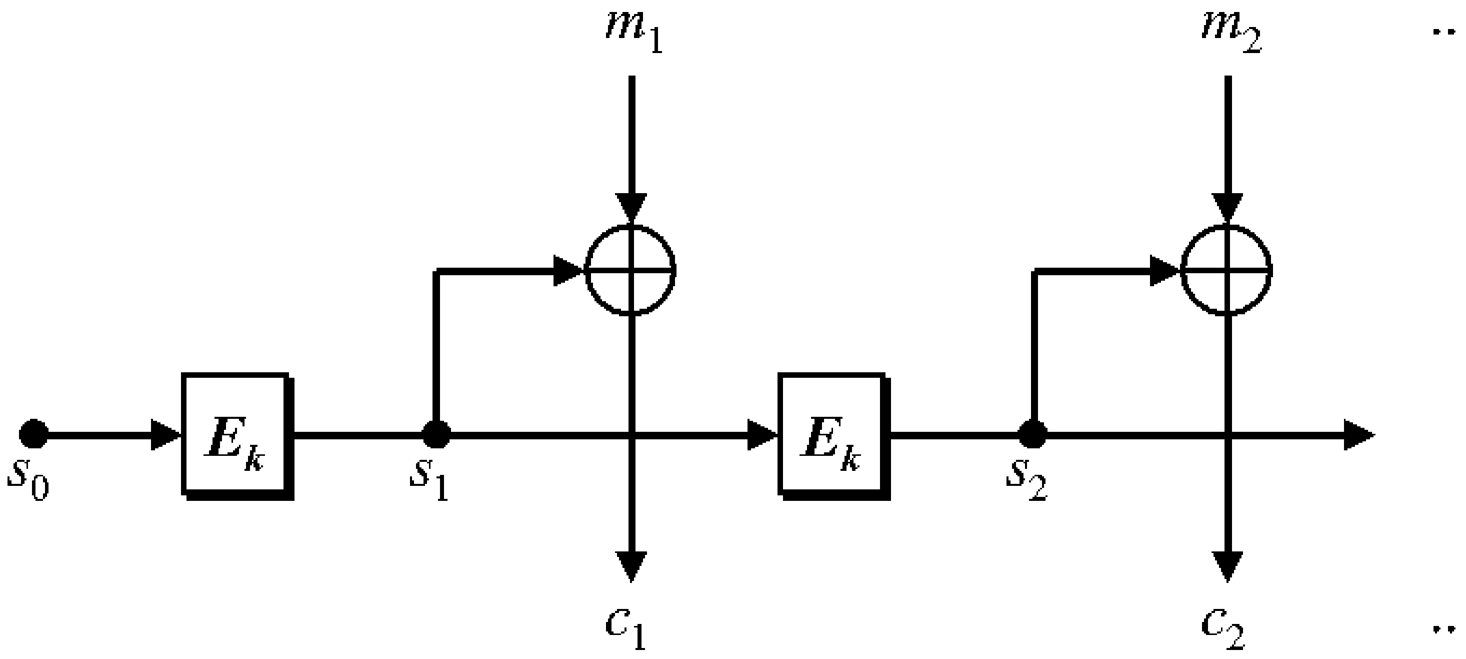
Decryption:



Properties of CBC

- Asynchronous: the receiver can start decrypting from any block in the ciphertext. ☺
- Errors in one *ciphertext* block propagate to the decryption of the next block (but that’s it). ☺
- Conceals plaintext patterns (same block $\Rightarrow$ different ciphertext blocks) ☺
  - But if IV is fixed, CBC does not hide not common *prefixes*
- No parallel implementation is known ☹
- Plaintext cannot be easily manipulated ☺
- Standard in most systems: SSL, IPSec, etc.

OFB Mode (Output FeedBack)



- An initialization vector s_0 is used as a "seed" for generating a sequence of "pad" blocks s_i. ($s_i=E_k(s_{i-1})$)
- Essentially a stream cipher
- s_0 can be sent in the clear.

Properties of OFB

- Synchronous stream cipher. I.e., the two parties must know s_0 and the current bit position. ☹
- The parties must synchronize the location they are encrypting/decrypting. ☹
- Errors in ciphertext do not propagate ☺
- Implementation:
  - Pre-processing is possible ☺
  - No parallel implementation known ☹
- Conceals plaintext patterns ☺
- Active attacks (by manipulating the plaintext) are possible ☹

Design of Block Ciphers

- More an art/engineering challenge than science. Based on experience and public scrutiny.
- *“Diffusion”:* each intermediate/output bit affected by many input bits
- *“Confusion”*: avoid structural relationships between bits

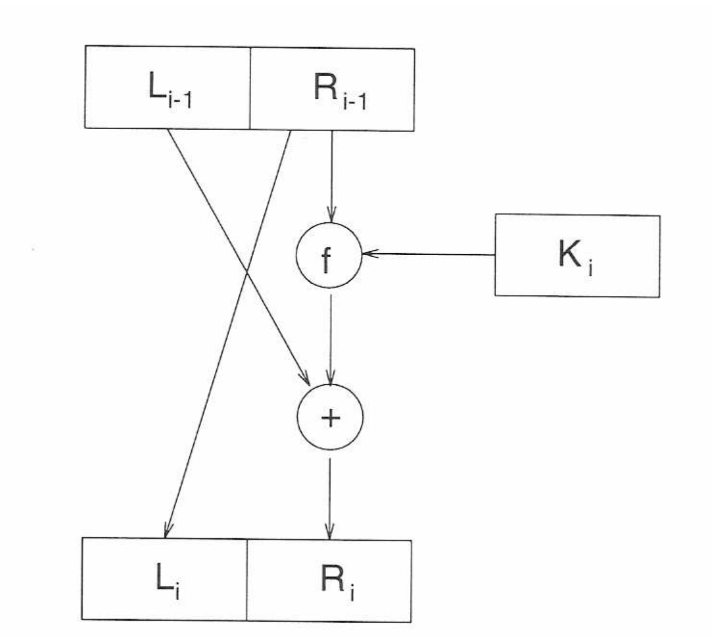
- Cascaded (round) design: the encryption algorithm is composed of iterative applications of a simple round

- A common round function: Feistel network

Feistel Networks

- Encryption:
- *Input:* P = L_{i-1} | R_{i-1}. $|L_{i-1}|=|R_{i-1}|$
  - $L_i = R_{i-1}$
  - $R_i = L_{i-1} \oplus F(K_i, R_{i-1})$
- Decryption?

- No matter which function is used as F, we obtain a permutation (i.e., F is reversible even if *f* is not).
- The same code/circuit, with keys in reverse order, can be used for decryption.
- Theoretical result [LubRac]: If F is a pseudo-random function then 4 rounds give a pseudo-random permutation



DES (Data Encryption Standard)

- A Feistel network encryption algorithm:
  - How many rounds?
  - How are the round keys generated?
  - What is F?

- DES (Data Encryption Standard)
  - Designed by IBM and the NSA, 1977.
  - 64 bit input and output
  - 56 bit key
  - 16 round Feistel network
  - Each round key is a 48 bit subset of the key
- Throughput ≈ software: 10Mb/sec, hardware: 1Gb/sec (in 1991!).
- Criticized for unpublished design *decisions* (designers did not want to disclose differential cryptanalysis).
- Linear cryptanalysis: about 2^{40} known plaintexts

What we've learned today

- Perfect security implies $|M| \leq |K|$
- Computational security
- Pseudo-randomness, Pseudo-random generator
- Block ciphers
- DES