

Introduction to Cryptography

Lecture 2

Benny Pinkas

Perfect Cipher

- What type of security would we like to achieve?
- “Given C , the adversary has no idea what M is”
 - Impossible since adversary might have a-priori information
- In an “ideal” world, the message will be delivered in a magical way, out of the reach of the adversary
 - We would like to achieve similar security
- Definition: a *perfect cipher*
 - $\Pr(\text{plaintext} = P \mid \text{ciphertext} = C) = \Pr(\text{plaintext} = P)$

Perfect Ciphers

- A simple criteria for perfect ciphers.
- Claim: The cipher is perfect if, and only if,
 $\forall m_1, m_2 \in M, \forall \text{cipher } c,$
 $\Pr(\text{Enc}(m_1)=c) = \Pr(\text{Enc}(m_2)=c). \quad (\text{homework})$
- Idea: Regardless of the plaintext, the adversary sees the same distribution of ciphertexts.
- Note that the proof cannot assume that the cipher is the one-time-pad, but rather only that $\Pr(\text{plaintext} = P \mid \text{ciphertext} = C) = \Pr(\text{plaintext} = P)$

Size of key space

- Theorem: For a perfect encryption scheme, the number of keys is at least the size of the message space.
- Proof:
 - Consider ciphertext C .
 - Must be a possible encryption of any plaintext m .
 - But, need a different key per message m .
- Corollary: Key length of one-time pad is optimal ☺

Computational security

- We should only worry about polynomial adversaries
- Idea: Generate a string which “looks random” to any polynomial adversary. Use it instead of a OTP.
- Looks random?
 - Fraction of bits set to 1 is $\approx 50\%$
 - Longest run of 0's is of length $\approx \log(n)$,
 - Is that sufficient?...
- Enumerating a set of statistical tests that the string should pass is not enough.

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Computational security – Pseudo-randomness

- Pseudo-random string: no *efficient* observer can *distinguish* it from a uniformly random string of the same length
- Motivation: *Indistinguishable objects are equivalent*
- The foundation of modern cryptography
- (t, ϵ) -Pseudo-random generator (PRG)
 - $G: \{0, 1\}^k \Rightarrow \{0, 1\}^m$ $|k| < |m|$, polynomially computable.
 - \forall adversary D running in time t ,
for $s \in_R \{0, 1\}^k$, $u \in_R \{0, 1\}^m$,
it holds that $\Pr(D(G(s)) \neq D(u)) < \epsilon$

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Pseudo-random generators

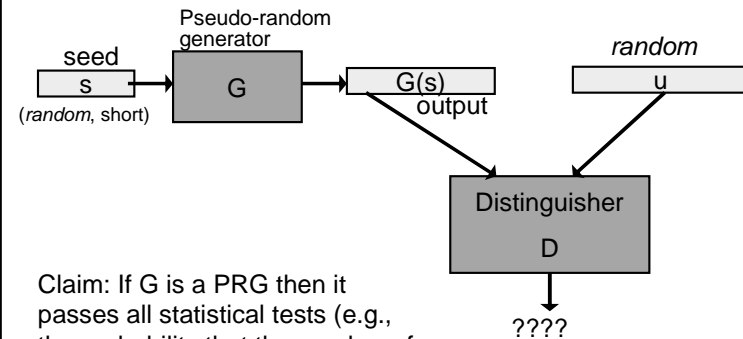
- Pseudo-random generator (PRG)
 - $G: \{0, 1\}^k \Rightarrow \{0, 1\}^m$ $|k| < |m|$, polynomially computable.
 - \forall polynomial time adversary D ,
for $s \in_R \{0, 1\}^k$, $u \in_R \{0, 1\}^m$,
it holds that $\Pr(D(G(s)) \neq D(u))$ is negligible
 - *Polynomial time*: running in time $t(n)$ s.t. \exists polynomial $p()$ for which $t(n) < p(n)$ for all large enough n
 - *Negligible*: the difference is a function $\epsilon(n)$ s.t. \forall polynomials $q()$, for all large enough n it holds that $\epsilon(n) < q(n)$

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Pseudo-random generator



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Using a PRG for Encryption

- Key: a (short) random seed $s \in \{0,1\}^{|k|}$.
- Message $m = m_1, \dots, m_{|m|}$.
- Encryption:
 - Use the output of the PRG as a one-time pad. Namely,
 - Generate $G(s) = g_1, \dots, g_{|m|}$
 - Ciphertext $C = g_1 \oplus m_1, \dots, g_{|m|} \oplus m_{|m|}$

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Using a PRG for Encryption: Security

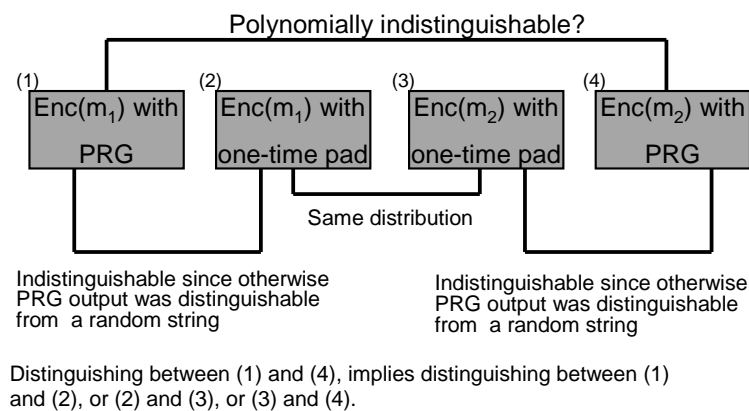
- One time pad:
 - $\forall m_1, m_2 \in M, \forall c$, the probability that c is an encryption of m_1 is equal to the probability that c is an encryption of m_2 .
 - I.e., $\forall m_1, m_2 \in M \forall c$, it is impossible to tell whether c is an encryption of m_1 or of m_2 .
- Security of pseudo-random encryption:
 - Show that $\forall m_1, m_2 \in M$, no *polynomial time* adversary can distinguish between the encryptions of m_1 and of m_2 .
- Proof by reduction: if one can break the security of the encryption (distinguish between encryptions of m_1 and of m_2), it can also break the security of the PRG (distinguish it from random).

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Proof of Security



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Symmetric systems used in practice

- Are not based on computational problems
- Are (usually) not proven secure by reductions
- Are designed for specific input and key lengths
- Are very efficient
- Stream ciphers
 - Meant to implement a pseudo-random generator
 - Usually used for encryption in the same way as OTP
 - Examples: A5, RC4, SEAL.
 - Require *synchronization*

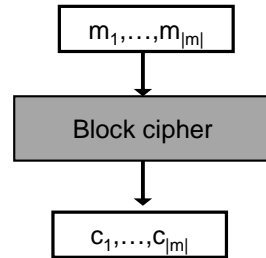
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Block Ciphers

- Plaintexts, ciphertexts of fixed length, $|m|$. Usually, $|m|=64$ or $|m|=128$ bits.
- The encryption algorithm E_k is a *permutation* over $\{0,1\}^{|m|}$, and the decryption D_k is its inverse.
- Ideally, use a *random* permutation. Instead, use a *pseudo-random* permutation, keyed by a key k .
- Encrypt/decrypt whole blocks of bits
 - Might provide better encryption by simultaneously working on a block of bits
 - Error propagation: one error in ciphertext affects whole block
 - Delay in encryption/decryption
- Different modes of operation

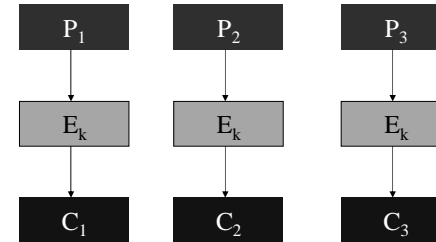


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ECB Encryption Mode (Electronic Code Book)



Namely, encrypt each plaintext block separately.

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Properties of ECB

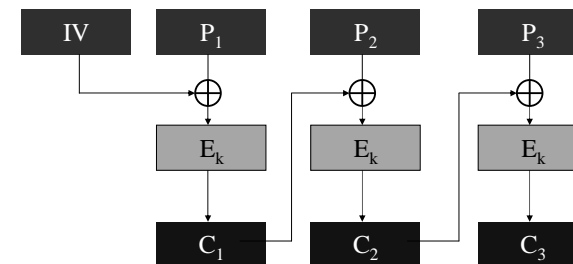
- Simple and efficient
- Parallel implementation is possible
- Does not conceal plaintext patterns
 - $\text{Enc}(P_1, P_2, P_1, P_3)$
- Active attacks are possible (plaintext can be easily manipulated by removing, repeating, or interchanging blocks).

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CBC Encryption Mode (Cipher Block Chaining)



Previous *ciphertext* is XORed with current *plaintext* before encrypting current block.

An initialization vector *IV* is used as a “seed” for the process. *IV* can be transmitted in the clear (unencrypted).

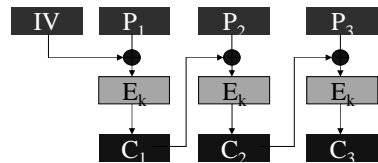
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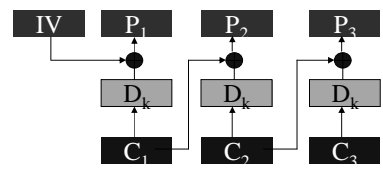
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CBC Mode

Encryption:



Decryption:



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Properties of CBC

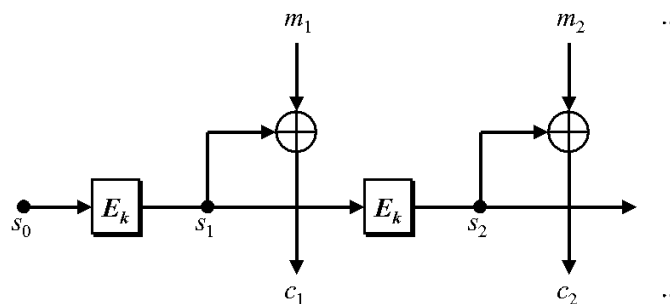
- Asynchronous: the receiver can start decrypting from any block in the ciphertext.
- Errors in one *ciphertext* block propagate to the decryption of the next block (but that's it).
- Conceals plaintext patterns (same block \rightarrow different ciphertext blocks)
 - But if IV is fixed, CBC does not hide not common prefixes
- No parallel implementation is known
- Plaintext cannot be easily manipulated.
- Standard in most systems: SSL, IPSec, etc.

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OFB Mode (Output FeedBack)



- An initialization vector s_0 is used as a "seed" for generating a sequence of "pad" blocks s_i . ($s_i = E_k(s_{i-1})$)
- Essentially a stream cipher
- s_0 can be sent in the clear.

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Properties of OFB

- Synchronous stream cipher. I.e., the two parties must know s_0 and the current bit position.
- The parties must synchronize the location they are encrypting/decrypting.
- Errors in ciphertext do not propagate
- Implementation:
 - Pre-processing is possible
 - No parallel implementation known
- Conceals plaintext patterns
- Active attacks (by manipulating the plaintext) are possible

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Design of Block Ciphers

- More an art/engineering challenge than science. Based on experience and public scrutiny.
- “*Diffusion*”: each intermediate/output bit affected by many input bits
- “*Confusion*”: avoid structural relationships between bits
- Cascaded (round) design: the encryption algorithm is composed of iterative applications of a simple round
- A common round function: Feistel network

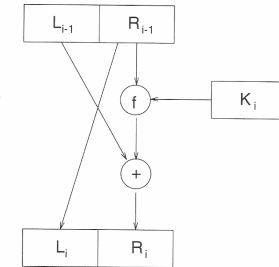
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Feistel Networks

- Encryption:
- Input: $P = L_{i-1} \parallel R_{i-1}$, $|L_{i-1}| = |R_{i-1}|$
 - $L_i = R_{i-1}$
 - $R_i = L_{i-1} \oplus F(K_i, R_{i-1})$
- Decryption?
- No matter which function is used as F , we obtain a permutation (i.e., F is reversible).
- The same code/circuit, with keys in reverse order, can be used for decryption.
- Theoretical result [Luby-Rackoff]: If F is a pseudo-random function then 4 rounds give a pseudo-random permutation



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DES (Data Encryption Standard)

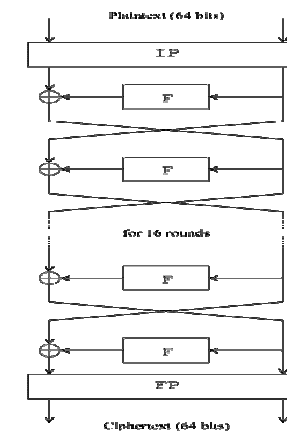
- A Feistel network encryption algorithm:
 - How many rounds?
 - How are the round keys generated?
 - What is F ?
- DES (Data Encryption Standard)
 - Designed by IBM and the NSA, 1977.
 - 64 bit input and output
 - 56 bit key
 - 16 round Feistel network
 - Each round key is a 48 bit subset of the key
- Throughput \approx software: 10Mb/sec, hardware: 1Gb/sec (in 1991!).
- Criticized for unpublished design *decisions* (designers did not want to disclose differential cryptanalysis).
- Linear cryptanalysis: about 2^{40} known plaintexts

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DES diagram

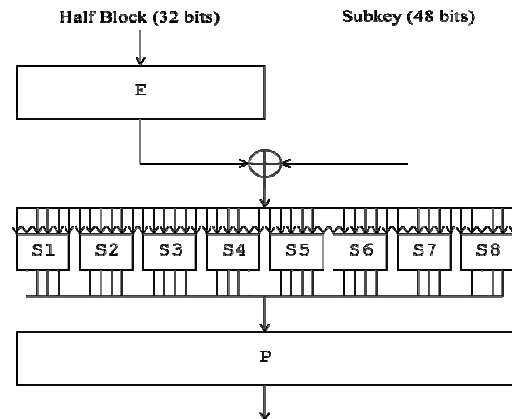


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DES F functions



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Double DES

- DES is out of date due to brute force attacks on its short key (56 bits)
- Why not apply DES twice with two keys?
 - Double DES: $\text{DES}_{k_1, k_2} = E_{k_2}(E_{k_1}(m))$
 - Key length: 112 bits
- But, double DES is susceptible to a meet-in-the-middle attack, requiring $\approx 2^{56}$ operations and storage.
 - Compared to brute force attack, requiring 2^{112} operations and $O(1)$ storage.



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Meet-in-the-middle attacks

- Meet-in-the-middle attack
 - $c = E_{k_2}(E_{k_1}(m))$
 - $D_{k_2}(c) = E_{k_1}(m)$
- The attack:
 - Input: (m, c) for which $c = E_{k_2}(E_{k_1}(m))$
 - For every possible value of k_1 , generate and store $E_{k_1}(m)$
 - For every possible value of k_2 , check if $D_{k_2}(c)$ is in the table
 - Might obtain several options for (k_1, k_2) . Check them or repeat the process again with a new (m, c) pair.
- The attack is applicable to any iterated cipher

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Triple DES

- $3\text{DES}_{k_1, k_2} = E_{k_1}(D_{k_2}(E_{k_1}(m)))$
- Why use $\text{Enc}(\text{Dec}(\text{Enc}(\)))$?
 - Backward compatibility: setting $k_1 = k_2$ is compatible with single key DES
- Only two keys
 - Effective key length is 112 bits
 - Why not use three keys? There is a meet-in-the-middle attack with 2^{112} operations
- Provides good security. Widely used. Less efficient.

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AES (Advanced Encryption Standard)

- Design initiated in 1997 by NIST
 - Goals: improve security and software efficiency of DES
 - 15 submissions, several rounds of public analysis
 - The winning algorithm: Rijndael
- Input block length: 128 bits
- Key length: 128, 192 or 256 bits
- Multiple rounds (10, 12 or 14), but does not use a Feistel network

What we've learned today

- Perfect security implies $|M| \leq |K|$
- Computational security
- Pseudo-randomness, Pseudo-random generator
- Block ciphers
- DES, AES
- Meet in the middle attack